Segmenting Images Corrupted by Correlated Noise*

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Abstract

Image segmentation is fundamental to many image analysis problems. It aims to partition a digital image into a set of non–overlapping homogeneous regions. This paper briefly describes a new segmentation procedure which is designed to segment images corrupted by correlated *noise. This procedure is based on Rissanen's minimum description length principle.*

1 Introduction

One popular approach to segmenting an image based on greyvalue is to approximate the image by a 2D piecewise smooth function and define the segmentation by the discontinuity points of the 2D function. In this paper images that can be well modelled by 2D piecewise constant functions (PCFs) corrupted by additive Gaussian noise will receive primary attention. The main contribution here is the proposal of a new segmentation procedure which is designed to segment such images when the noise is *correlated*.

The proposed segmentation procedure is based on Rissanen's minimum description length (MDL) principle [4] and consists of two components: i) an MDL–based criterion which the "best" segmentation is defined as its minimizer and ii) a fast merging algorithm which attempts to locate this minimizer. To the best of our knowledge, this is the first time that, for the piecewise smooth function modelling approach to the problem of image segmentation, the effect of noise correlation has been taken into account.

The MDL principle has been previously applied to the problem of image segmentation, e.g. see [1], [2], [5] and references given therein. The key difference here is that, all such previous work only considered *independent* noise.

2 Problem Formulation

As suggested before, we model images by 2D PCFs corrupted by additive Gaussian noise. Let

$$\boldsymbol{f} = (f_1, \ldots, f_n)^T$$

be a discrete version of an arbitrary 2D PCF with k constant regions and n pixels. Denote the greyvalue of the jth region as μ_j, $j = 1, \ldots, k$, and write

$$\boldsymbol{\mu} = (\mu_1, \ldots, \mu_k)^T.$$

Then $\boldsymbol{f}$ can be represented by:

$$f_i = \sum_{j=1}^{k} \mu_j I_{\{i \in r_j\}}, \qquad i = 1, \ldots, n,$$

where "$i \in r_j$" means "the ith pixel is in the jth region", and I_E is the indicator function for the event E. Notice that we have chosen to use single indexing rather than double indexing for labelling pixel coordinates. Also notice that in our formulation for $\boldsymbol{f}$, region boundaries are composed of horizontal and vertical "edges" between pixels. For convenience, we define

$$\boldsymbol{\Omega} = \{r_1, \ldots, r_k\}$$

as the set of all region boundaries, i.e. $\boldsymbol{\Omega}$ defines a partition of the image.

Our aim is, given an observed image

$$\boldsymbol{y} = (y_1, \ldots, y_n)^T$$

satisfying

$$y_i = f_i + \epsilon_i, \qquad i = 1, \ldots, n,$$

where the ϵ_i's are zero mean Gaussian random errors, to obtain an estimate $\hat{\boldsymbol{f}}$ for $\boldsymbol{f}$, and define our segmentation of the image by the discontinuity points of $\hat{\boldsymbol{f}}$. We will apply the MDL principle to obtain such an estimated/fitted 2D PCF $\hat{\boldsymbol{f}}$.

Briefly, the MDL principle defines the best fitted model as the one that enables the best encoding of the data, so that the data can be transmitted in the most economical way. In the present situation, the data is the image to be segmented and a fitted model is simply a fitted 2D PCF (i.e. a segmentation) of the image. That is, the best fitted model/segmentation is the one that produces the shortest code length of the data/image. A typical code length formula usually consists of two parts: i) the code length for specifying a fitted model plus ii) the code length for specifying the data "conditioning on" the fitted model (i.e. the residuals).

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3 MDL–Based Segmentation for Independent Noise

Now, for convenience, we state the result regarding the code length formula for encoding an image corrupted by *independent* noise (see [3]). We need some notation to proceed. Suppose a fitted 2D PCF is given. Let $\hat{k}$ be its estimated number of regions, $\hat{\mathbf{\Omega}} = \{\hat{r}_1, \ldots, \hat{r}_{\hat{k}}\}$ be the set of all its estimated region boundaries (i.e. discontinuity points), and $\hat{\boldsymbol{\mu}} = (\hat{\mu}_1, \ldots, \hat{\mu}_{\hat{k}})^T$ be its estimated region greyvalues. If a_j and b_j are respectively the area (in terms of number of pixels) and perimeter (in terms of number of pixel edges) of the jth region of $\hat{\mathbf{\Omega}}$, then it can be shown that the following MDL criterion

$$\mathrm{MDL}_{\text{IND}}(\hat{k}, \hat{\mathbf{\Omega}}) = \hat{k} \log n + \frac{\log 3}{2} \sum_{j=1}^{\hat{k}} b_j + \frac{1}{2} \sum_{j=1}^{\hat{k}} \log a_j + \frac{n}{2} \log(\mathrm{RSS}_{\hat{k}}/n) \qquad (1)$$

is an approximation to the desirable code length formula. Here

$$\mathrm{RSS}_{\hat{k}} = \sum_{i=1}^{n} (y_i - \hat{f}_i)^2$$

is the residual sum of squares with $\hat{f}_i$ defined by

$$\hat{f}_i = \sum_{j=1}^{\hat{k}} \hat{\mu}_j I_{\{i \in \hat{r}_j\}}, \; \hat{\mu}_j = \frac{1}{a_j} \sum_{s \in \hat{r}_j} y_s, \; i = 1, \ldots, n. \quad (2)$$

Therefore one can define a segmentation of an image corrupted by *independent* noise as the minimizer of $\mathrm{MDL}_{\text{IND}}(\hat{k}, \hat{\mathbf{\Omega}})$.

4 MDL–Based Segmentation for Correlated Noise

Because of various computational difficulties we propose to use the following 3–parameter autocovariance function (acvf) to approximate the acvf of the correlated noise corrupting the image being segmented:

$$\mathrm{acv}(r, c) = A \exp\left(-\alpha_r \Delta_r |r| - \alpha_c \Delta_c |c|\right), \qquad (3)$$
$$r = \frac{-n_r}{2}, \ldots, \frac{n_r}{2} - 1, \quad c = \frac{-n_c}{2}, \ldots, \frac{n_c}{2} - 1,$$

where Δ_r and Δ_c are the sampling intervals in the row and column directions respectively, $n_r \times n_c$ is the dimension of the image (i.e. $n_r n_c = n$), and $A > 0$, $\alpha_r > 0$ and $\alpha_c > 0$ are parameters. Here we assume both n_r and n_c are even, and images are observed in the square $[-1, 1) \times [-1, 1)$. Thus $\Delta_r = 2/n_r$ and $\Delta_c = 2/n_c$.

It can be shown that an MDL criterion similar to (1) but for correlated noise generated from acvf (3) is:

$$\mathrm{MDL}_{\text{COR}}(\hat{k}, \hat{\mathbf{\Omega}}) = \hat{k} \log n + \frac{\log 3}{2} \sum_{j=1}^{\hat{k}} b_j + \frac{1}{2} \sum_{j=1}^{\hat{k}} \log a_j + \frac{n}{2} \log\left[\hat{A}(1 - \exp(-2\hat{\alpha}_r \Delta_r))(1 - \exp(-2\hat{\alpha}_c \Delta_c))\right], \quad (4)$$

where $\hat{A} = \mathrm{RSS}_{\hat{k}}/n$, $\hat{\alpha}_r$ and $\hat{\alpha}_c$ are estimates of A, α_r and α_c respectively. Expressions for $\hat{\alpha}_r$ and $\hat{\alpha}_c$ are:

$$\hat{\alpha}_r = \sum_{(i,j) \in S_r} (\hat{e}_i - \hat{e}_j)^2 (2\hat{A}\Delta_r |S_r|)^{-1}$$

and

$$\hat{\alpha}_c = \sum_{(i,j) \in S_c} (\hat{e}_i - \hat{e}_j)^2 (2\hat{A}\Delta_c |S_c|)^{-1}.$$

Here $\hat{e}_i = y_i - \hat{f}_i$ is the ith residual, S_r and S_c are sets defined as:

$$S_r = \{(i, j) \; : \; \text{pixel } j \text{ is the immediate right neighbour of pixel } i\},$$
$$S_c = \{(i, j) \; : \; \text{pixel } j \text{ is the immediate bottom neighbour of pixel } i\},$$

and $|S_r|$ and $|S_c|$ are the numbers of elements in S_r and S_c respectively.

We propose to choose our segmentation for images corrupted by *correlated* noise as the minimizer of the criterion $\mathrm{MDL}_{\text{COR}}(\hat{k}, \hat{\mathbf{\Omega}})$. We also developed a practical merging algorithm to approximate this minimizer. Notice that $\mathrm{MDL}_{\text{COR}}(\hat{k}, \hat{\mathbf{\Omega}})$ converges to $\mathrm{MDL}_{\text{IND}}(\hat{k}, \hat{\mathbf{\Omega}})$ when both $\hat{\alpha}_r$ and $\hat{\alpha}_c$ approach infinity.

5 Experimental Results

We tested both the independent and correlated segmentation procedures with images obtained from adding different correlated noise to the true but unknown image displayed in Fig. 1. The correlated noise was generated from the following three different acvfs:

1. *separable exponential* (a member of (3)):

$$\mathrm{acv}(r, c) = 10 \exp\left(-10\Delta_r |r| - 5\Delta_c |c|\right),$$

2. *nonseparable exponential*:

$$\mathrm{acv}(r, c) = 10 \exp\left[-5\sqrt{(\Delta_r r)^2 + 4(\Delta_c c)^2}\right],$$

3. *bivariate Gaussian*:

$$\mathrm{acv}(r, c) = 20 \exp\left[-20\left\{(\Delta_r r)^2 + 4(\Delta_c c)^2\right\}\right].$$

Realizations of some of such generated noise patterns (or textures) are given in Fig. 2. Notice that acvfs 2 and 3 do not satisfy our assumption on the underlying true but unknown structure of acvf; see (3).

Corresponding segmentation results are displayed in Figs. 3 and 4. One empirical conclusion is that, ignoring the presence of positively correlated noise would result in a serious oversegmentation (of course this is not necessary true if the correlation is so weak that the noise can be treated as independent). Another empirical conclusion is that, even though our acvf structure assumption was wrong for acvfs 2 and 3, the proposed correlated noise segmentation procedure did give reasonable results.

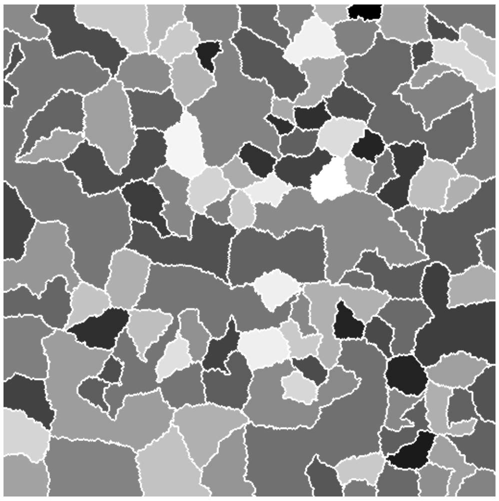

Figure 1: True but unknown test image with true boundaries overlaid (in white).

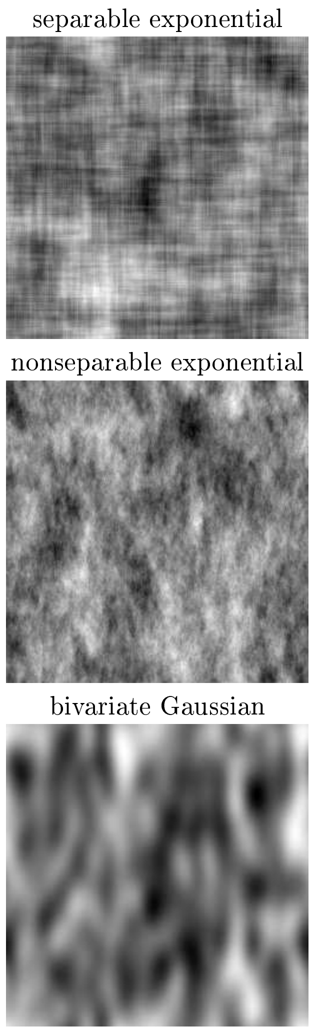

Figure 2: Various noise patterns/textures.

References

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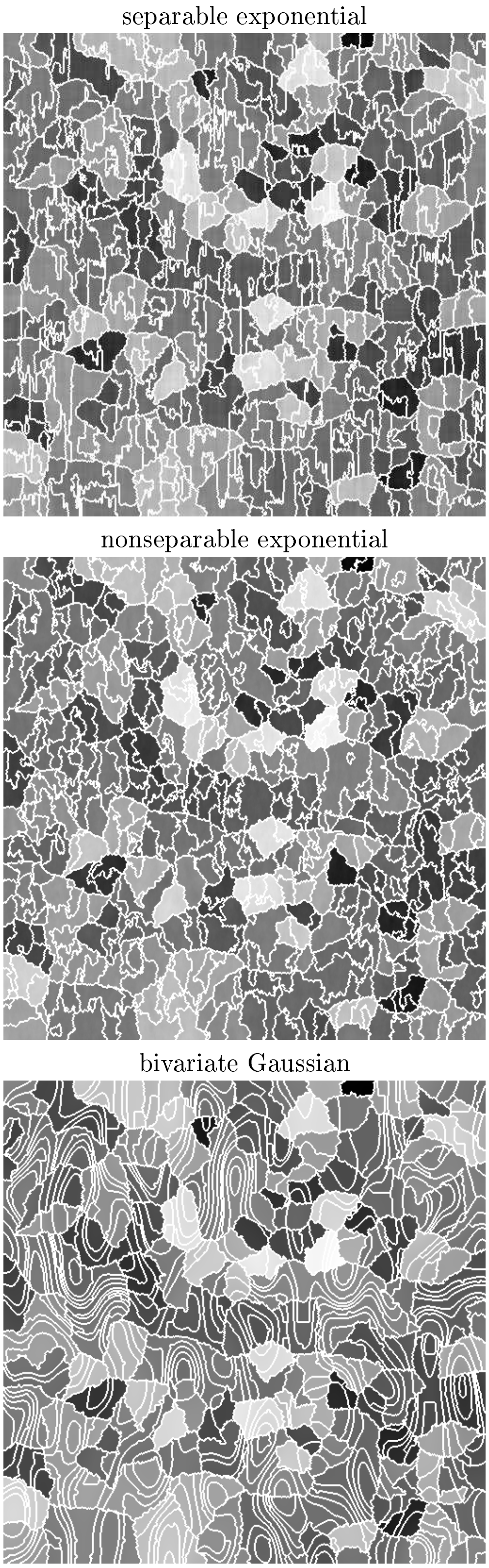

Figure 3: Results of independent noise segmentation procedure.

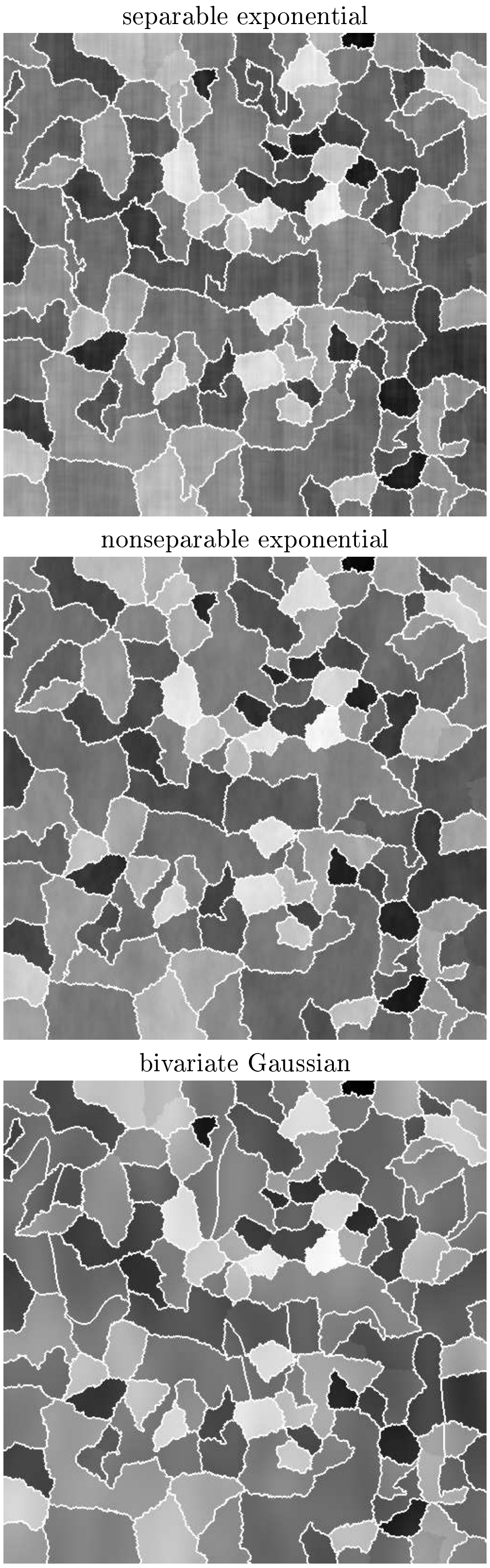

Figure 4: Results of correlated noise segmentation procedure.