Automatic Reconnection of Linear Segments by the Minimum Description Length Principle

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Abstract

The automatic reconnection of linear segments is a problem often encountered in image analysis. This article proposes a procedure for performing this task. Tuning parameters of the proposed procedure can either be chosen manually, or chosen automatically by a method developed in this note. This automatic method is based on the minimum description length principle. The procedure is applied to some real images.

Keywords: edge linking, line detection, object recognition, Vision

1 Introduction

One common problem encountered in image analysis is the detection of long narrow features captured in images. These features can correspond to edges of larger objects, or can themselves constitute the objects to segment (for example as in Fig. 2). A possible approach for tackling this feature detection problem is first to detect local linear segments of the target features and then to reconnect these detected linear segments. In previous work [1], we have developed a procedure for reconnecting such detected linear segments. This has the advantage that it is quite fast and easy to implement, and can also be used for reconnecting overlapping objects. However, in order to apply this procedure, the user has to specify two "tuning parameters": λ and t (defined below). Quite often, these parameters are chosen subjectively, and hence require human interaction. To minimize such intervention, the (λ, t) are often fixed a-priori for a large number of images, which implies that some compromise is made.

This paper presents our recent work on developing an automatic method for choosing (λ, t) in an adaptative way, meaning that (λ, t) are chosen automatically for each image, depending on its content. This automatic method is based on Rissanen's minimum description length (MDL) principle [2]. In the following sections, we first review the reconnection procedure presented in [1], and then present our automatic parameter selection method.

2 Reconnection Procedure of Lee & Talbot (1995)

We suppose that, for a given image, a set of linear segments have been detected and we want to reconnect them. By "two linear segments are reconnected" we mean a straight line is drawn from one extremity of one segment to one extremity of the other segment. We shall call such a straight line a *reconnection*. The procedure starts with assigning a score to each of all possible reconnections. Such a score is given by a *score function* f:

$$f(\alpha, \beta, l) \quad = \quad \lambda\sqrt{\tfrac{(2\pi-\alpha-\beta)}{2\pi}} + \frac{l(1-\lambda)}{diag},$$

where, as illustrated in Fig. 1(a), α and β are angles (less than or equal to π) between the possible reconnection and each of the two involved segments, l is the length of the possible reconnection, *diag* is the length of the diagonal of the image and λ is an adjustable parameters within the range [0, 1]. Notice that the range of f is [0, 1], and that f is a decreasing function of both $(2\pi - \alpha - \beta)$ and l.

The first term of f measures how well the two linear segments are aligned while the second term measures how close the two involved segment extremities are. The adjustable parameter λ can be used to control the relative weights of these two measures. If two segments are well aligned and close to each other, it is likely that they belong to the same long feature in the original image, and in this case f gives a score close to zero.

After all the scores have been calculated, those possible reconnections with a score less than a pre–chosen threshold t are performed, subject to the constraint that no two reconnections share a common segment extremity. If more than one possible reconnection is assigned a score less than the threshold and shares one common extremity, only the reconnection with the smallest score will be performed.

3 Automatic Choice of Parameters by the Minimum Description Length Principle

We now present a new MDL–based method for choosing (λ, t). With a proper setup, the MDL principle defines the "best" combination of (λ, t) as the one that enables the best encoding (or compression) of the linear segments. Therefore, in order to apply the MDL principle to tackle the problem, we first need to construct a code length expression which calculates the amount of space that is required to store a set of reconnected linear segments resulted from a given (λ, t) combination. We need some notation to proceed (illustrated in Fig. 1(b)): a *reconnected object* R consists of a series of linear segments and reconnections (the linear segments are linked together by the reconnections). We denote the linear segments in R by $s_0, s_1, \dots, s_n$ and the reconnections by $r_0, r_1, \dots, r_{n-1}$, where n is the number of reconnections in R. We also denote the length of s_i and r_i as $|s_i|$ and $|r_i|$ respectively. Let p and a_0 respectively be the position of the only free extremity and the slope of s_0. Also let α_i and β_i respectively be the smaller angles between s_i and r_i, and between r_i and s_{i+1}, $i = 0, \dots, n-1$.

If we assume the following:

- A1: a_0 is uniformly distributed in $[0, 2\pi[$
- A2: $|r_0|, \dots, |r_{n-1}|$ are independent and identically distributed (i.d.d.) exponential random variables (r.v.'s) with unknown mean μ, i.e.:

  $$P(|r_i| = l) = \frac{1}{\mu}\exp(\frac{-l}{\mu}),$$

  for all i, where $P(E)$ denotes the probability of E.
- A3: $(\pi - \alpha_i)$ and $(\pi - \beta_i)$ are i.d.d. truncated exponential r.v.'s with unknown mean ν and truncation at π for all i , i.e.:

  $$P((\pi - \alpha_i) = \theta) = \frac{\frac{1}{\nu}\exp(\frac{-\theta}{\nu})}{1 - \exp(\frac{-\pi}{\nu})}.$$

A2 implies that we encourage short reconnections and A3 that we prefer segments which are well aligned. Then we can show (see [3]) that the code length $C(R)$ required to encode R can be well approximated by (up to an additive constant):

$$C(R) = \log N + 3n + n\log\bar{l} + \log n + 2n\left[\log\left\{1 - \exp(\tfrac{-\pi}{\hat{\nu}})\right\} + \log\hat{\nu} + \tfrac{\bar{\theta}}{\hat{\nu}}\right], \tag{1}$$

where $\hat{\nu} = \bar{\theta} + \pi/\left\{\exp(\pi/\bar{\theta}) - 1\right\}$, $\bar{l}$ and $\bar{\theta}$ are the means of the l_i's and θ_i's respectively. and N is the total number of pixels in the image.

The above code length expression can be generalized to the code length expression for encoding an arbitrary set of linear segments linked into p different pieces (i.e. p different reconnected objects). Suppose that the jth reconnected object R_j has n_j reconnections and let $\bar{l}_j$ be the mean length of these n_j reconnections. Also let $\bar{\theta}_j$ be the "mean angles" of R_j (defined similarly to $\bar{\theta}$ above). Then the total code length for encoding p reconnected objects is:

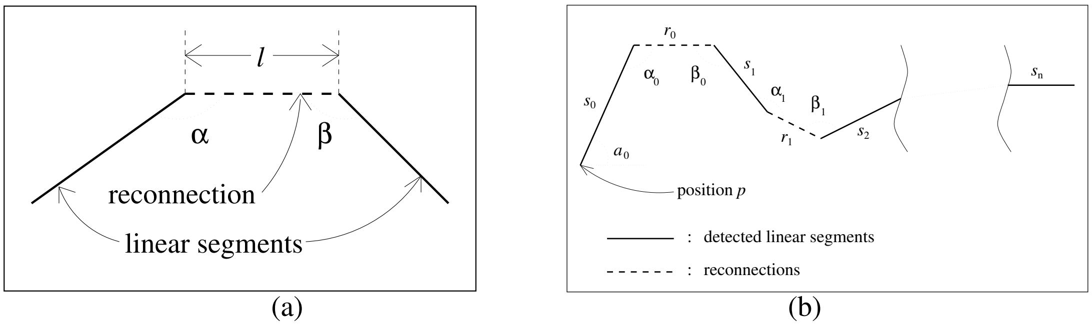

Figure 1: Definitions for: (a) linear segments and reconnections. (b) reconnected objects.

$$\sum_{j=0}^{p-1} C(R_j) =$$
$$p \log N + 3\sum_{j=0}^{p-1} n_j + \sum_{j=0}^{p-1} n_j \log \bar{l}_j + \sum_{j=0}^{p-1} \log n_j$$
$$+\, 2\sum_{j=0}^{p-1} n_j \left[\log \left\{ 1 - \exp(\frac{-\pi}{\hat{\nu}_j}) \right\} + \log \hat{\nu}_j + \frac{\bar{\theta}_j}{\hat{\nu}_j} \right] \tag{2}$$

with

$$\hat{\nu}_j = \bar{\theta}_j + \pi \left\{ \exp(\frac{\pi}{\bar{\theta}_j}) - 1 \right\}^{-1} .$$

With the help of the above expression, the parameters (λ, t) can be chosen automatically with a grid search approach: for a given set of detected linear segments, we first apply the reconnection procedure with different combinations of (λ, t), and obtain a candidate linear segment reconstruction for each of the (λ, t) combinations. Then for all these candidate reconstructions we apply expression (2) to calculate their corresponding code length. Finally the candidate reconstruction with the smallest code length is chosen as the final reconstruction.

This approach allows the optimal combination of (λ, t) in the MDL sense to be chosen adaptatively on each different image, depending on its content.

4 Experimental results

4.1 A first fibre image

Fig. 2 shows an application where the goal was to measure the length of the two long thin fibres present in the image in Fig. 2(a). Notice that some portions of these fibres are barely visible and that the background of the image is very noisy. We adopted a "segment-detect-reconnect" approach, described as follows:

We first obtained an initial segmentation of the thin fibres using a classical morphological white top-hat operator (see [4]) together with a series of linear openings. Then a linear segment detector (see [5]) was used to improve the location of white ridge points in the images, which yielded markers for the location of the thin fibres. The resulting detected linear segments are shown in Fig 2(b).

We applied the proposed reconnection procedure with MDL–based choice of (λ, t) to the image in Fig. 2(c), which is the correct result.

4.2 A second fibre image

In this example, we are now trying to segment and reconnect the large fibres in Fig. 3(a). As in the previous section, we adopted the "segment–detect–reconnect" approach: we first obtained an initial egmentation of the fibres (see Fig. 3(b)), then a morphological skeleton operation [6] was applied to the initial segmentation, and the linear segments displayed in Fig. 3(d) were extracted from the resulting skeleton shown in Fig. 3(c). These linear segments were then reconnected by the proposed reconnection procedure, with parameters chosen automatically by the MDL–based method. The reconnected segments are displayed in Fig. 3(e). Observe that they were all correctly reconnected. Using these reconnected segments we obtained the final segmentation displayed in Fig. 3(f).

5 Discussion

The most interesting points of the proposed approach is *(a)* its adaptability and *(b)* its apparent lack of need to adjust a-priori parameters. Experimentally, we have found that the reconnections obtained with this method look quite reasonable, but we have found no way of proving that this will always be the case. In fact it is quite easy to find examples where even a human operator would have trouble finding the "correct" reconnections.

We have found that *(a)* and *(b)* are a major improvement over more traditional edge-linking methods. However, *(b)* is only an appearance: the parameters of the method are actually hidden in the model chosen for coding the reconnection. Different probability distributions would lead to different results.

We have chosen to find the minimum coding length in essence by "trial and error" (grid search). This is acceptable in the applications shown, because the number of possible reconnections is actually quite low. In more complex problems, a more sophisticated minimization technique would have to be employed.

6 Conclusion

In this paper we have presented a procedure for reconnecting linear segments. This procedure is adaptative in the sense that tuning parameters which govern the correctness of the reconnection are chosen automatically for each image. The principle which guides the adaptative method is the minimum description length principle.

We have shown that this procedure is easy to implement and yields reasonable results for practical problems.

References

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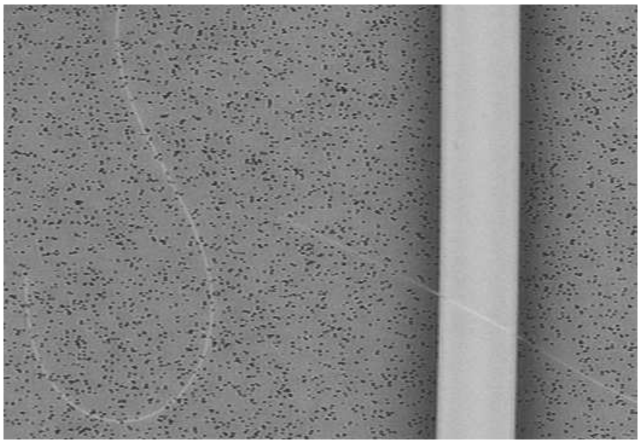
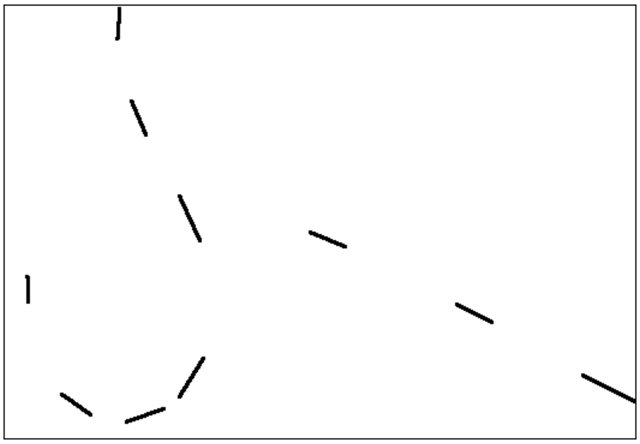
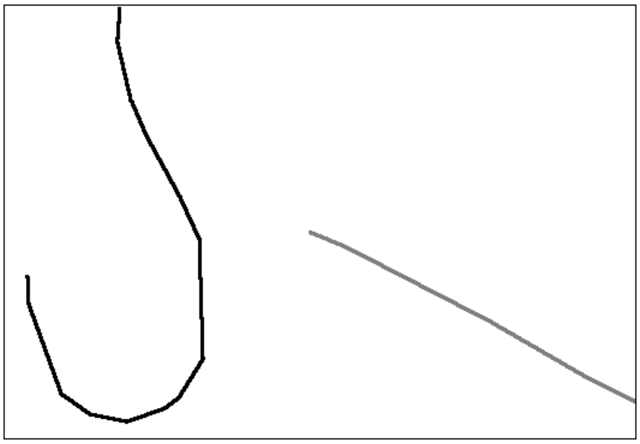


(a) glass fibres

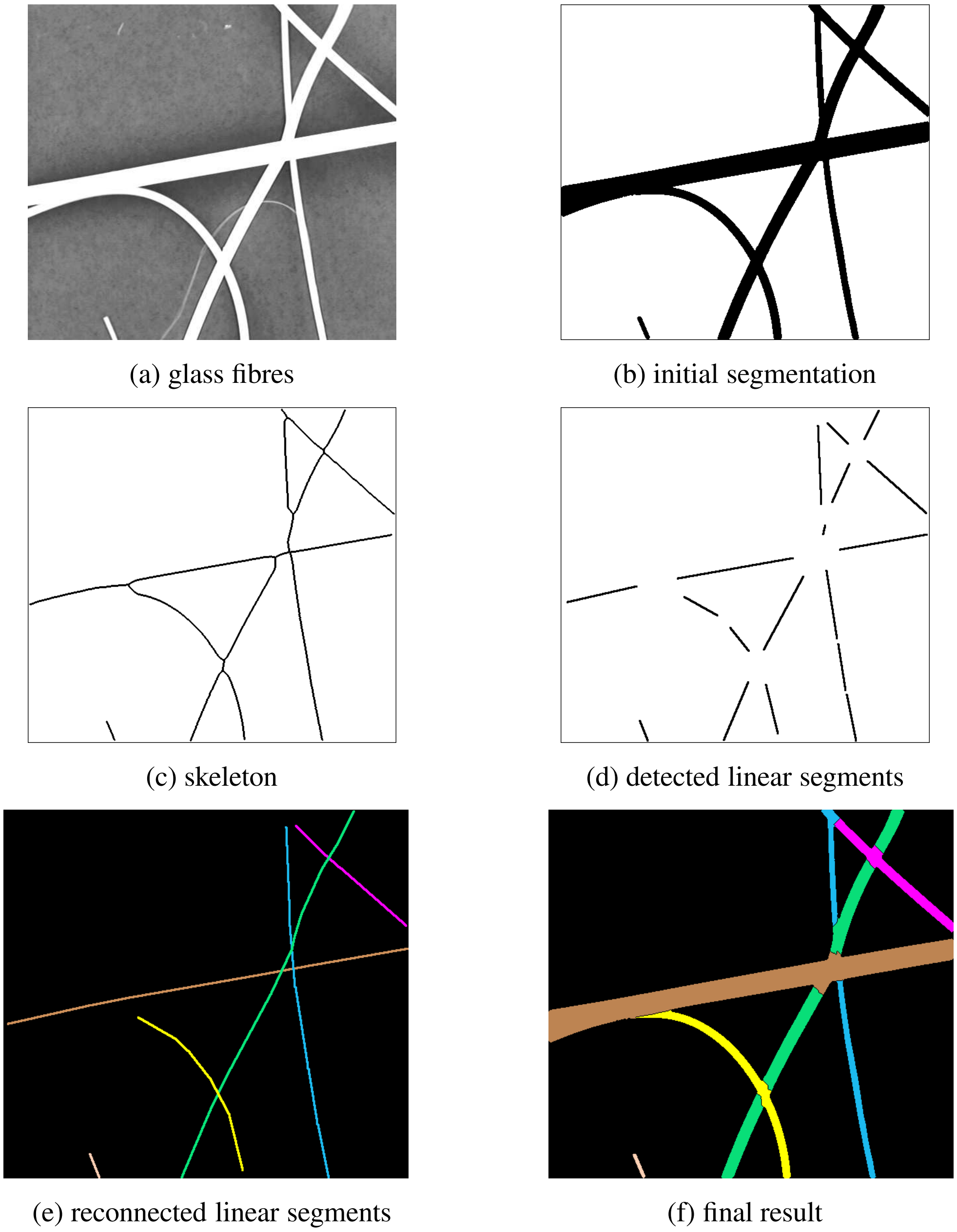


(b) detected linear segments



(c) final result

Figure 2: An image of glass fibres.



(a) glass fibres (b) initial segmentation (c) skeleton (d) detected linear segments (e) reconnected linear segments (f) final result

Figure 3: An image of glass fibres.