Supplemental Material to "Fréchet Estimation of Time-Varying Covariance Matrices From Sparse Data, With Application to the Regional Co-Evolution of Myelination in the Developing Brain"

Analysis Using Complete Data Set

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Below we show the figures corresponding to Figures 2–5 obtained be utilizing the complete data set of 440 observations, including repeated measurements for approximately half of the $n = 223$ subjects. Bandwidths were chosen using the same procedure described in Section 3.2, and are reported in the captions.

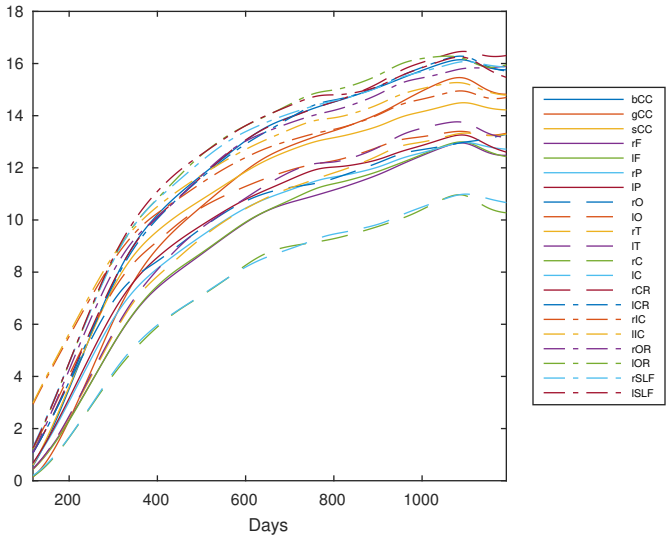
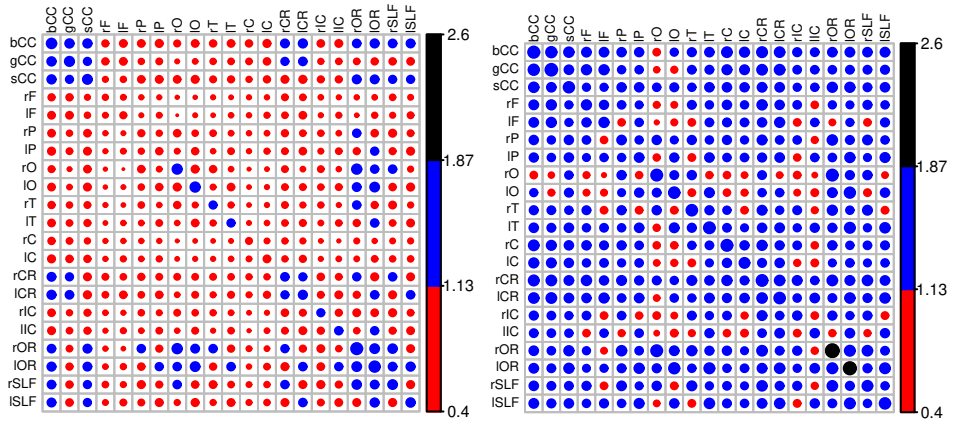


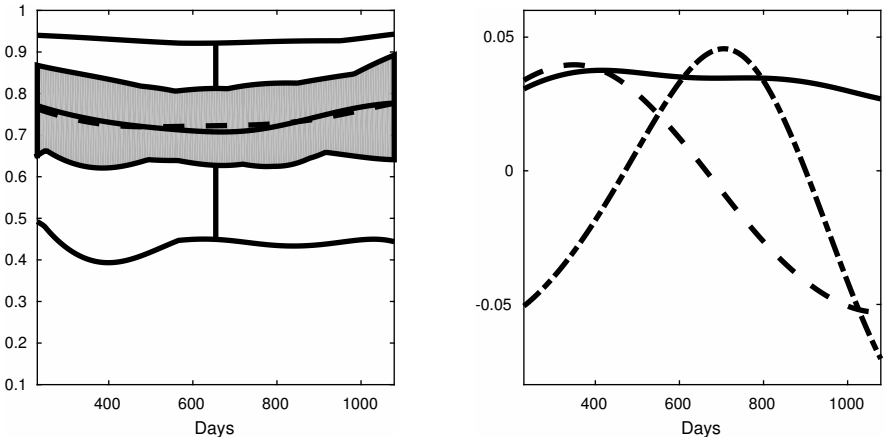
Fig 7: Estimated mean functions for all 21 regions using a bandwidth of 53 days.



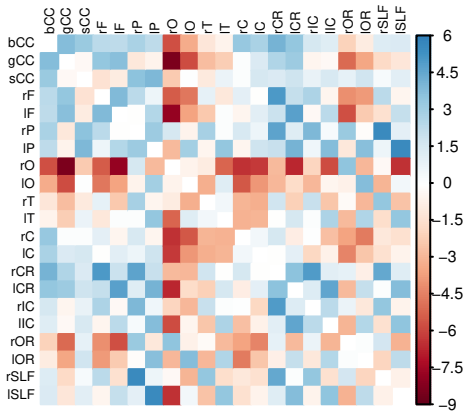
(a) $x = 250$ days (b) $x = 500$ days

(c) $x = 750$ days (d) $x = 1000$ days

Fig 8: Estimated MWF covariance matrices at four distinct ages x (in days), using bandwidth $h = 160$ days. Circle size indicates magnitude of covariance (all are positive), and color indicates a discretization of the covariance values for easier visual comparison.

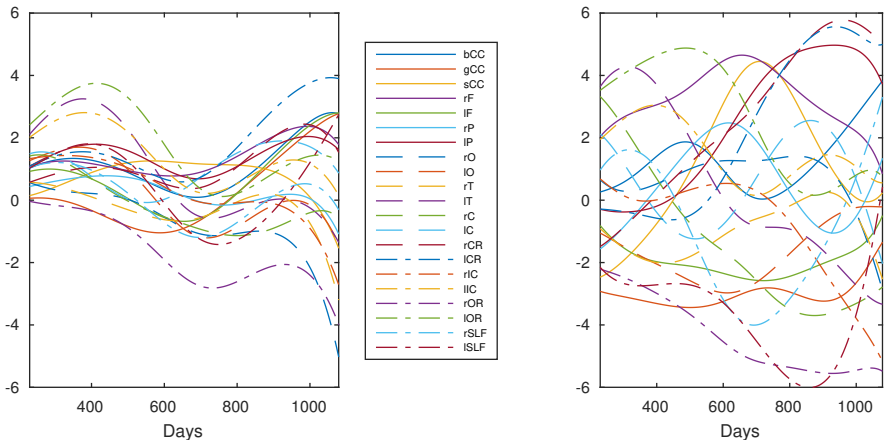


(a) (b)



(c)

Fig 9: (a) Functional boxplot, including mean (dashed curve), for correlation curves between the 21 regions. (b) First (solid, FVE = 90.1%), second (dashed, FVE = 6.15%) and third (dot-dash, FVE = 2.76%) eigenfunction representing the variability in the collection of estimated correlation curves. (c) First principal scores for all region pairs.



(a) $\hat{\Gamma}$ (b) $\hat{\beta}_{\hat{\lambda}_{\text{opt}}}$

Fig 10: Estimates $\hat{\Gamma}(x)$ and $\hat{\beta}_{\hat{\lambda}_{\text{opt}}}(x)$ corresponding to the linear model (3.2) for regressing ELC score on myelin water fraction levels. The optimal ridge parameter chosen here was $\hat{\lambda}_{\text{opt}} = 0.11$.