4.18 By Result 4.11 we know that the maximum likelihood estimates of \( \mu \) and \( \tau \) are \( \bar{x} = [4, 6]' \) and

\[
\frac{1}{n} \sum_{j=1}^{n} (x_j - \bar{x}) (x_j - \bar{x})' = \frac{1}{4} \begin{bmatrix}
\frac{3}{2} & 0 & 0 & 0 \\
0 & \frac{3}{2} & 0 & 0 \\
0 & 0 & \frac{3}{2} & 0 \\
0 & 0 & 0 & \frac{3}{2}
\end{bmatrix}
\] = \frac{1}{4} \begin{bmatrix}
2 & 1 \\
1 & 6
\end{bmatrix}
\]

4.19 a) By Result 4.7 we know that \( \frac{(X_1 - \mu)'(X_1 - \mu)}{\tau^2} \sim \chi^2_6 \)

b) From (4-23), \( \bar{x} - N(\mu, \frac{1}{20} \tau) \). Then \( \bar{x} - \mu \sim N(0, \frac{1}{20} \tau) \) and finally \( \sqrt{20} (\bar{x} - \mu) \sim N(0, \tau) \)

c) From (4-23), \( \chi^2 \) has a Wishart distribution with 19 d.f.

4.21 (a) \( X \) is distributed \( N(\mu, n^{-1} \Sigma) \)

(b) \( X_1 - \mu \) is distributed \( N(0, \Sigma) \) so \( (X_1 - \mu)'n^{-1}(X_1 - \mu) \) is distributed as chi-square with \( p \) degrees of freedom.

(c) Using Part a),

\[
(X - \mu)'(n^{-1}\Sigma)^{-1}(X - \mu) = n(X - \mu)'\Sigma^{-1}(X - \mu)
\]

is distributed as chi-square with \( p \) degrees of freedom.

(d) Approximately distributed as chi-square with \( p \) degrees of freedom. Since the sample size is large, \( \Sigma \) can be replaced by \( S \).

4.22 a) We see that \( n = 75 \) is a sufficiently large sample (compared with \( p \)) and apply Result 4.13 to get \( \sqrt{n}(X - \mu) \) is approximately \( N_p(0, \frac{1}{n} \tau) \) and that \( \bar{x} \) is approximately \( N_p(\mu, \frac{1}{n} \tau) \).

b) By (4-28) we conclude that \( \sqrt{n}(X - \mu)'S^{-1}(X - \mu) \) is approximately \( \chi^2_p \).
4.24 (a) Q-Q plots for sales and profits are given below. Plots not particularly straight, although Q-Q plot for profits appears to be "straighter" than plot for sales. Difficult to assess normality from plots with such a small sample size (n = 10).

(b) The critical point for n = 10 when α = .10 is .9351. For sales, \( r_0 = .940 \) and for profits, \( r_0 = .968 \). Since the values for both of these correlations are greater than .9351, we cannot reject normality in either case.