Outline of Lecture
#6

Sta 13 B/C/D

P. 1/2

Ref. Ch. 5, Sec. 5.1, 5.2.

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Now we discuss one frequently used discrete prob. distn. — Binomial distn..

Binomial Experiment: An experiment is known to be Binomial if it has the following characteristics:

1. There is a fixed no. of trials, n ['Trial' is any performance of an expt.]
2. Each trial results in either of two outcomes, viz. 'success' and 'failure'.
3. p, the prob. of success, is same for each trial.
4. All the trials are independent.
5. We are interested in the no. of successes (X) observed during n trials.

- Binomial random variable (X): The no. of successes under Binomial setting is known as a Binomial r.v. which can take the values $0, 1, 2, \ldots, n$.
- Binomial distribution: Probability distribution of a Binomial r.v. is known as binomial distribution.

Let X = no. of successes out of n trials,

n = no. of trials

p = prob. of success for each trial $(0 < p < 1)$

$q = 1 - p$.

Possible values of X are $0, 1, 2, \ldots, n$. Then under Binomial setting, X is said to follow a Binomial distribution with parameters n and p (X is $B(n, p)$).

Binomial probabilities are given by the following expression:

Prob. of k successes in n trials,

$$\boxed{P(X = k) = C^n_k \cdot p^k \cdot q^{n-k} = \frac{n!}{k!\,(n-k)!} \cdot p^k \cdot (1-p)^{n-k}},$$

$k = 0, 1, 2, \ldots n$; $q = 1 - p$,

where

$$C^n_k = \frac{n!}{k!\,(n-k)!}.$$

(the symbol $n!$ is called 'n-factorial' and represents $n! = 1.2.3.\cdots n$ and $0! = 1$).

Ex. 5.1 – 5.6 ... 192

Cont... P.2

- Binomial mean and standard deviation.

Mean, $\mu = np$

Variance, $\sigma^2 = np(1-p)$

s.d., $\sigma = \sqrt{np(1-p)}$.

Ex. Vide Ex. 5.7, p. 193

Note: Binomial distr. is symmetric if $p = \frac{1}{2}$,
skewed to the right if $p < \frac{1}{2}$, skewed to the left if $p > \frac{1}{2}$.

Example: Let the prob. of a patient recovering from a certain disease be 0.75. Find the probability that (i) no patient, (ii) one patient, (iii) 2 patients, (iv) 3 patients, (v) all of 4 patients, (vi) at most 3, (vii) at least 3, (viii) 2 or 3, (ix) 3 or more, (x) more than 3, (xi) less than 3 will recover?
(xii) What is the expected no. of recoveries? What is its s.d.?

Solution: Probability distribution of no. of recoveries

K	$C^n_K = \frac{n!}{K!(n-K)!}$	$p^K(1-p)^{n-K}$	$P(X=K)$ = Col. 2 X Col. 3
0	$C^4_0 = 1$	$(0.25)^4 = 0.0039$	0.004
1	$C^4_1 = 4$	$(0.75)(0.25)^3 = 0.0117$	0.047
2	$C^4_2 = 6$	$(0.75)^2 . (0.25)^2 = 0.0352$	0.211
3	$C^4_3 = 4$	$(0.75)^3 . (0.25) = 0.1055$	0.422
4	$C^4_4 = 1$	$(0.75)^4 = 0.3164$	0.316
TOTAL	—	—	1.000

X = no. of recoveries
Here X is Binomial with parameters
$n = 4$, $p = 0.75$

(i) P(no patient will recover) = $P(X=0) = \boxed{0.004}$

(ii) $P(X=1) = \boxed{0.047}$, (iii) $P(X=2) = \boxed{0.211}$, (iv) $P(X=3) = \boxed{0.422}$,

(v) $P(X=4) = \boxed{0.316}$, (vi) P(at most 3 will recover) $= P(X \le 3) = P(0) + P(1) + P(2) + P(3)$
$= \boxed{0.684}$

(vii) P(at least 3 recover) $= P(X \geq 3) = P(X=3) + P(X=4) = \boxed{0.738}$

(viii) P(2 or 3 recover) $= P(X=2) + P(X=3) = \boxed{0.633}$

(ix) P(3 or more recover) $= P(X \geq 3) = \boxed{0.738}$

(x) P(more than 3 recover) $= P(X > 3) = P(X=4) = \boxed{0.316}$

(xi) P(less than 3 recover) $= P(X < 3) = P(X=0) + P(X=1) + P(X=2) = \boxed{0.262}$

(xii) Expected no. of recoveries, $\mu = n.p = 4 \times 0.75 = \boxed{3}$

Standard deviation, $\sigma = \sqrt{np(1-p)} = \sqrt{4 \times 0.75 \times 0.25} = \sqrt{0.75} = \boxed{0.866}$

(vi) Alternatively,
$P(X \le 3)$
$= 1 - P(X=4)$
$= \boxed{0.684}$