

STA13-B
Elementary Statistics
Fall 2007

Lecture 11

Instructor: Katie Pollard

Announcements

- No homework this week
- OH: M 10am, W 8am
- Midterm 1
 - Friday, October 26
 - Multiple choice
 - Bring a UCD2000 scantron
 - Formula sheet and study guide on website

Chapter 6

- Estimating probabilities
- Mutually exclusive outcomes
- Additive Rule

Probability of an Event

- Probability of an event B is the **sum** of the probabilities of all the simple events in B.

Example: $P(\text{throw} > 0 \text{ pts}) = P(1 \text{ pt}) + P(2 \text{ pts})$

- If the simple events are equally likely,

$$P(B) = \frac{\text{\# of simple events in B}}{\text{\# of simple events total}}$$

Example: $P(\text{even die role}) = 3/6 = 1/6 + 1/6 + 1/6$
 $= P(2) + P(4) + P(6)$

How to Estimate Probabilities

- If the sample space is too large or the events too complex, it is not possible to list all the simple events.

Example: GDP of 100 random countries

- Even when the simple events can be listed, it is not possible to directly compute the probability of an event whenever the simple events are not equally likely.

Example: Shooting free throws

Estimation Methods

$P(A) \approx$ proportion of times A occurs

Two methods for computing probabilities:

1. **Empirically** (i.e. from a large sample)

- Need to repeat the experiment many times
- The conditions must be identical each time

2. **By simulation** (i.e. from a fake large sample)

- Need to repeat the fake experiment many times
- The simulated experiment must very similar to the real situation of interest

Mutually Exclusive

Two outcomes are **mutually exclusive** if they cannot occur simultaneously.

- Simple events are always mutually exclusive.
- Events that share no simple events are also.

Example: Roll 2 dice

{14} and {42} **YES**

{both even} and {one is 6} **NO**

{sum > 10} and {at least one 1} **YES**

Simultaneous Events

Experiment: roll 2 dice

■ = both even, □ = one is 6

1,1	2,1	3,1	4,1	5,1	6,1
1,2	■ 2,2	3,2	■ 4,2	5,2	■ 6,2
1,3	2,3	3,3	4,3	5,3	6,3
1,4	■ 2,4	3,4	■ 4,4	5,4	■ 6,4
1,5	2,5	3,5	4,5	5,5	6,5
□ 1,6	■ 2,6	3,6	■ 4,6	5,6	■ 6,6

Additive Rule

For mutually exclusive outcomes, the probability that at least one occurs is the sum.

- Probability of **at least one** is denoted by "or"

\cup = union = or

- Probability of **both** is denoted by "and"

\cap = intersection = and

$$P(A \text{ or } B) = P(A) + P(B), \text{ if } P(A \text{ and } B) = 0$$

$$P(A \cup B) = P(A) + P(B), \text{ if } P(A \cap B) = 0$$

DNA Sequence Changes

Example: Probability one DNA base changes

- Sample space: $\{A \rightarrow A, A \rightarrow C, A \rightarrow T, A \rightarrow G, C \rightarrow A, C \rightarrow C, C \rightarrow T, C \rightarrow G, T \rightarrow A, T \rightarrow C, T \rightarrow T, T \rightarrow G, G \rightarrow A, G \rightarrow C, G \rightarrow T, G \rightarrow G\}$
- Event1: $\{\text{ends A}\} = \{A \rightarrow A, C \rightarrow A, T \rightarrow A, G \rightarrow A\}$
- Event2: $\{\text{ends T}\} = \{A \rightarrow T, C \rightarrow T, T \rightarrow T, G \rightarrow T\}$
- $P(\text{ends A or T}) = P(A \rightarrow A) + P(C \rightarrow A) + P(T \rightarrow A) + P(G \rightarrow A) + P(A \rightarrow T) + P(C \rightarrow T) + P(T \rightarrow T) + P(G \rightarrow T)$

General Additive Rule

The probability of any one event is the sum of the probabilities that they occur individually minus the probability of their both occurring.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

Example: DNA changes

$$\begin{aligned} P(\text{no change or end A}) &= P(A \rightarrow A) + P(C \rightarrow C) + \\ &P(T \rightarrow T) + P(G \rightarrow G) + P(A \rightarrow A) + P(C \rightarrow A) + P(T \rightarrow A) \\ &+ P(G \rightarrow A) - P(A \rightarrow A) \end{aligned}$$

Mutually Exclusive Events

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

If A and B mutually exclusive $P(A \text{ and } B)=0$

$$P(A \text{ or } B) = P(A) + P(B) - 0$$

$$= P(A) + P(B) \quad \leftarrow$$

Additive rule
for mutually
exclusive events

Example: DNA changes

$$P(\text{ends T or ends A}) = P(A \rightarrow T) + P(C \rightarrow T) + P(T \rightarrow T) + P(G \rightarrow T) + P(A \rightarrow A) + P(C \rightarrow A) + P(T \rightarrow A) + P(G \rightarrow A) - \cancel{P(\text{ends T and ends A})}$$

Complement of an Event

- A^c = all simple events not in the event A .
- A and A^c are mutually exclusive, so

$$P(A \text{ and } A^c) = 0$$

- Since either A and A^c must occur,

$$P(A \text{ or } A^c) = P(A) + P(A^c) = 1$$

- Hence, $P(A) = 1 - P(A^c)$

