STATISTICS 13

Lecture 10

Jan. 26, 2007

Review

- Linear regression
  - Formula

$$y = a + bx,$$

$$b = r\frac{s_y}{s_x}$$

$$a = \bar{y} - b\bar{x}$$

  - Interpretation of regression result
  - Regression effect

Probability: An Introduction

- Probability is used as a mathematical tool to understand/describe random phenomena, chance variation and uncertainty
- Example: emission tests on motor vehicles. Four repeated measurements on the same car

HC (gm/mile)	13.8	18.3	32.2	32.5
CO (gm/mile)	118	149	232	286

There is substantial variation. How to make a precision measure?

- Applications: genetics; kinetics; quality control; finance, etc

Basic Concepts: Experiments

- Probability is used as a model/tool for situations for which outcomes occur randomly, such situations are called experiments, and the set of all possible outcomes is called the sample space.
- Examples :
  - Toss a coin
  - Measure the blood pressure
  - Draw a poker hand from a card-deck
  - Throw two dice

Basic Concepts: Simple Events

- A simple event is the outcome of a single repetition of a random experiment
- Example :

  Experiment : toss a coin once

  Simple events : Head (H), Tail (T)
- One and only one simple event can occur when an experiment is performed once
- One can assign a probability to each simple event in the sample space

Basic Concepts: Sample Space

- The set of all possible simple events of an experiment is called the sample space (usually denoted by S) corresponding to that experiment
- Example :
  - Experiment : Toss two coins once
  - Sample space :

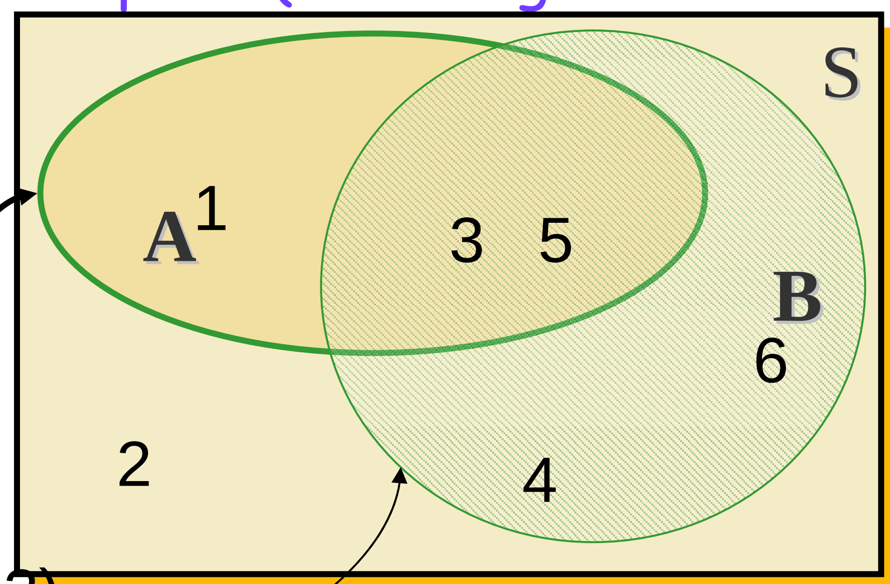
Basic Concepts: Events

- An event (usually denoted by E, F, A, B, etc) is a collection of simple events. In another word, a subset of the sample space
- Any subset of the sample space (including the empty set) is an event
- Example :
  Throw a die

S = {1,2,3,4,5,6}

A : (An odd number)
= {1,3,5}

B : (A number > 2)
= {3,4,5,6}



Basic Concepts: Mutually Exclusive

- Two events A and B are mutually exclusive if whenever one of them occurs the other cannot occur
- Example : Throw a die

  A: observe an odd number

  B: observe a number greater than 2

  C: observe a 6

  D: observe a 3

Probability: Relative Frequency View

- How often does "an event" occur ?

Relative frequency = *f/n*

- As sample size *n* becomes "large"

Relative frequency ⟶ Probability

Probability of an Event: Properties

- Probability of an event is a number between 0 and 1
- Sum of the probabilities of all the simple events in S (sample space) equals to 1
- Probability of any event *A*, denoted by P(*A*) is the sum of the probabilities of all the simple events that constitute *A*
- P(*A*) = 0 means *A* never occurs
- P(*A*) = 1 means *A* always occurs

Examples

- Toss a fair coin twice. What is the probability of observing at least one head ?
- In this case, all simple events are equally likely.

First toss	Second toss	E_i	$P(E_i)$
H	H	HH	1/4
	T	HT	1/4
T	H	TH	1/4
	T	TT	1/4

Sample Space with Equally Likely Simple Events

- If the simple events are equally likely, then for event *A*

$$P(A) = \frac{n_A}{N} = \frac{\text{number of simple events in A}}{\text{total number of simple events}}$$

- Need counting rules to find n_A and N

The Multiplicative Rule

- If an experiment is performed in two stages, with ***m*** possibilities for the first stage and ***n*** possibilities for the second stage, then there are ***mn*** ways to accomplish the experiment.
- This rule is easily extended to ***k*** stages, with the number of ways equal to $n_1 n_2 n_3 \ldots n_k$

Example : Toss two coins ;

Total number of simple events =

Example: Urn problem

- Urn problem: An urn contains three red balls and two yellow balls. Two balls are randomly chosen without replacement, what is the probability that the two balls are the same color?
- Answer:

  -there are N= simple events in the sample space, and they are all equally likely (due to random selection); n_A

  -event A={two balls are the same color}

Example: Urn problem (Cont.)

- How about the two balls are chosen with replacement?
- Answer:

  - there are N= simple events in the sample space, and they are all equally likely.

  - event A={two balls are the same color} ={r1r1,r1r2,r1r3,r2r1,r2r2, r2r3,r3r1,r3r2,r3r3, y1y1,y1y2,y2y1,y2y2}, so number of simple events in A

  n_A =

Probability: As Proportion

- Example: In a genetics experiment, the researcher mated two Drosophila fruit flies and observed two traits: wing size and eye color of 300 offspring

	Wing Size	
Eye Color	Normal	Miniature
Normal	141	6
Vermillion	3	150

- One of these offspring is randomly selected and observed for the two traits. What is the sample space? What probability you should assign to each simple event in the sample space?
- Answer:

  -The sample space consists of four simple events: E1=normal wing and normal eye, E2=normal wing and vermillion eye, E3=miniature wing and normal eye, E4=miniature wing and vermillion eye

  -A reasonable assignment of probabilities to simple events is by proportion, so P(E1)=141/300=0.47, P(E2)=3/300=0.01, P(E3)=6/300=0.02, P(E4)=150/300=0.50

  -Note that all the properties of probabilities are satisfied, so this is a legitimate assignment
- What is the probability that the fly has normal wing size?
- Answer:

Counting: Permutations

- The number of ways you can choose r objects in order from *n* distinct objects

$$P_r^n = \frac{n!}{(n-r)!} = n(n-1)(n-2)\cdots(n-r+1)$$

where $n! = n(n-1)(n-2)...(2)(1)$ and $0! \equiv 1$

Example: Candidates

- Suppose we want to rank five candidates Adam, Bette, Carl, David, Elle in preference for a position, how many different rankings are possible?
- **Answer:**
- Suppose we can only choose two of them as possible candidates and then rank the two chosen persons, then how many possible rankings are there?
- **Answer:** (AB,AC,AD,AE,BA,BC,BD,BE,CA,CB,CD,CE,DA,DB,DC,DE,EA,EB,EC,ED)

Example : Cards

- 4 cards on the table : 2 3 4 5
- Draw cards one after another
- Win if the second card has higher denomination. What is the probability ?
- Simple events : a two card sequence (order of drawing is important); all equally likely
- Total number of events N =
- Event A ={the second card has higher denomination}=

Counting: Combinations

- The number of ways you can choose r objects without regarding order from *n* distinct objects

$$C_r^n = \frac{n!}{r!(n-r)!}$$

Example: Candidates

- Suppose we want to form a two-member committee from five candidates Adam, Bette, Carl, David, Elle , how many different committees are possible?
- Answer: Note that unlike the “ranking” problem, here the order of choice/selection is not important

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  --possible committees are:

  --what’s the difference compared to the ranking problem?

Example : Cards

- 5 cards on the table :
- How many different ways to select 3 cards to form a 3-card hand
- Order of the drawing is not important