Statistics 120 Spring Quarter 2008

Answers to First Midterm

1) (i) It is given that $P(A)=0.3$, $P(B)=0.5$, $P(A\cap B)=0.2$. Then $P(\text{at least one of A or B occurs}) = P(A\cup B) = P(A)+P(B)-P(A\cap B) = 0.3+0.5-0.2 = \underline{0.6}$.

(ii) $P(\text{none of A or B occurs}) = P(A^c \cap B^c) = P[(A\cup B)^c] = 1-P(A\cup B)$, so that $0.27 = 1-P(A\cup B)$, and $P(A\cup B) = 1-0.27 = \underline{0.73}$.

2) number letters numbers

(i) $10\times 26^3\times 10^3 = 26^3\times 10^4 = 17576\times 10{,}000 = \underline{175{,}760{,}000}$.

(ii) $10\times 26\times 25\times 24\times 10^3 = 15{,}600\times 10{,}000 = \underline{156{,}000{,}000}$.

(iii) $10\times 26\times 25\times 24\times 10\times 9\times 8 = 15{,}600\times 7{,}200 = \underline{112{,}320{,}000}$.

(iv) $1\times 23^3\times 10\times 1\times 10 = 12{,}167\times 100 = \underline{1{,}216{,}700}$.

3) (i) $x+\frac{1}{2}\geq 0 \Leftrightarrow x\geq -\frac{1}{2}$ which is true;

$\int_0^1 (x+\frac{1}{2})dx = \frac{x^2}{2}+\frac{1}{2}\cdot x\Big|_0^1 = \frac{1}{2}(1+1) = \frac{2}{2} = \underline{1}$.

(ii) $P(\frac{1}{2}<X<\frac{3}{4}) = \int_{1/2}^{3/4}(x+\frac{1}{2})dx = \frac{x^2}{2}+\frac{1}{2}\cdot x\Big|_{1/2}^{3/4} = \frac{1}{2}(x^2+x)\Big|_{1/2}^{3/4} =$

$\frac{1}{2}\left(\frac{9}{16}+\frac{3}{4}-\frac{1}{4}-\frac{1}{2}\right) = \frac{1}{2}\times\frac{9+12-4-8}{16} = \frac{9}{32} = \underline{0.28125}$.

4) With obvious notation:

(i) $P(S) = P(S \cap M) + P(S \cap F) = P(S|M)P(M) + P(S|F)P(F)$

$= (0.30 \times 0.45) + (0.25 \times 0.55) = 0.135 + 0.1375 = \underline{0.2725}.$

(ii) $P(F|S) = \dfrac{P(F \cap S)}{P(S)} = \dfrac{P(S|F)P(F)}{P(S)} = \dfrac{0.1375}{0.2725} = \dfrac{1375}{2725} \simeq \underline{0.5046}.$

5) In the first place, the assumptions imply that the events E_1, E_2, E_3 are independent. Then so are the events we get if any number of the three ~~events is re-~~placed by their complements. Then, we have:

(i) $P(F_3) = P(E_1^c \cap E_2^c \cap E_3^c) = P(E_1^c)P(E_2^c)P(E_3^c) = (1-p)^3,$

$P(F_2) = P[(E_1^c \cap E_2^c \cap E_3) \cup (E_1^c \cap E_2 \cap E_3^c) \cup (E_1 \cap E_2^c \cap E_3^c)]$

$= P(E_1^c \cap E_2^c \cap E_3) + P(E_1^c \cap E_2 \cap E_3^c) + P(E_1 \cap E_2^c \cap E_3^c)$

$= P(E_1^c)P(E_2^c)P(E_3) + P(E_1^c)P(E_2)P(E_3^c) + P(E_1)P(E_2^c)P(E_3^c)$

$= \underline{3(1-p)^2 p}.$

$P(F_1) = P[(E_1^c \cap E_2 \cap E_3) \cup (E_1 \cap E_2^c \cap E_3) \cup (E_1 \cap E_2 \cap E_3^c)]$

$= P(E_1^c \cap E_2 \cap E_3) + P(E_1 \cap E_2^c \cap E_3) + P(E_1 \cap E_2 \cap E_3^c)$

$= P(E_1^c)P(E_2)P(E_3) + P(E_1)P(E_2^c)P(E_3) + P(E_1)P(E_2)P(E_3^c)$

$= \underline{3(1-p)p^2},$

$P(F_0) = P(E_1 \cap E_2 \cap E_3) = P(E_1)P(E_2)P(E_3) = \underline{p^3}.$

(ii) $P(F_3) = (0.03)^3 = \underline{0.000027}$

$P(F_2) = 3(0.03)^2(0.97) = \underline{0.002619}$

$P(F_1) = 3(0.03)(0.97)^2 = \underline{0.084681}$

$P(F_0) = (0.97)^3 = \underline{0.912673}.$