Department of Statistics

Statistics 120 Spring Quarter 2008

SAMPLE Second Midterm: Answers

1) (i) $(Y > y)$ means that in the interval $(0, y]$ no events occurred. The probability of this happening is: $e^{-\lambda y}\frac{(\lambda y)^0}{0!} = e^{-\lambda y}$. So, $P(Y > y) = e^{-\lambda y}$.

(ii) From $e^{-\lambda y} = 1 - F_Y(y)$, we get by differentiation (w.r. to y): $-\lambda e^{-\lambda y} = -f_Y(y)$, so that $f_Y(y) = \lambda e^{-\lambda y}$ $(y > 0)$.

(iii) It is the Negative Exponential with parameter λ.

2) (i) $\int_0^1 c(1-x^2)\,dx = c.x\Big|_0^1 - \frac{c}{3}.x^3\Big|_0^1 = c - \frac{c}{3} = \frac{2c}{3} = 1$, and $\underline{c = 3/2}$.

(ii) $P(X > \frac{1}{2}) = c.x\Big|_{1/2}^1 - \frac{c}{3}.x^3\Big|_{1/2}^1 = c(1-\frac{1}{2}) - \frac{c}{3}(1-\frac{1}{8}) = \frac{5c}{24} = \frac{5}{24} \times \frac{3}{2} = \underline{\frac{5}{16}}$.

(iii) Since $P(X > \frac{1}{2}) = \frac{5}{16}$, it follows that $P(X < \frac{1}{2}) = \frac{11}{16}$, so that $\underline{\frac{1}{2}}$ is the $\frac{11}{16}$ $(=0.6875)$-th quantile of X.

3) (i) $f_X(x) = \int_x^1 8xy\,dy = 8x.\frac{y^2}{2}\Big|_x^1 = \underline{4x(1-x^2)}$, $0 < x < 1$.

(ii) $f_{Y|X}(y|x) = \frac{8xy}{4x(1-x^2)} = \underline{\frac{2y}{1-x^2}}$, $0 < x < y < 1$.

(iii) $E(Y|X=x) = \int_x^1 y.\frac{2y}{1-x^2}\,dy = \frac{2}{1-x^2}.\frac{y^3}{3}\Big|_x^1 = \underline{\frac{2(1-x^3)}{3(1-x^2)}}$, $0 < x < 1$.

(iv) $E(Y|X=1/2) = \frac{2(1-\frac{1}{8})}{3(1-\frac{1}{4})} = \frac{7/4}{9/4} = \underline{\frac{7}{9}}$.

4) (i) $EX = (-2 \times \frac{1}{4}) + (-1 \times \frac{1}{4}) + (1 \times \frac{1}{4}) + (2 \times \frac{1}{4}) = \underline{0}.$

(ii) $XY = X^3$. Thus:

xy	-8	-1	1	8
$f_{XY}(xy)$	$1/4$	$1/4$	$1/4$	$1/4$

5) (i) $EX^2 = (4 \times 1/4) + (1 \times 1/4) + (1 \times 1/4) + (4 \times 1/4) = \underline{5/2} = \underline{Var(X)},$

(ii) Y: 1 4 ; $1/2$ $1/2$; $EY = (1 \times \frac{1}{2}) + (4 \times \frac{1}{2}) = \underline{5/2},$

$EY^2 = (1 \times \frac{1}{2}) + (16 \times \frac{1}{2}) = \underline{17/2},$

$Var(Y) = EY^2 - (EY)^2 = \frac{17}{2} - \frac{25}{4} = \underline{11/4}.$

(iii) $Var(2X + 6Y) = 4Var(X) + 36Var(Y) + 12\,Cov(X,Y)$

$= 4Var(X) + 36\,Var(Y)$

$= 4 \times \frac{5}{2} + 36 \times \frac{11}{4} = 10 + 99 = 109.$