Statistics 120

First Midterm: Answers

1) (i) $P(A\cup B)=P(A)+P(B)-P(A\cap B)$, so that $P(A\cap B)=$
$P(A)+P(B)-P(A\cup B)=0.3+0.6-0.7=0.2.$

(ii) $P(A\cap B^c)=P[A-(A\cap B)]=P(A)-P(A\cap B)=0.3-0.2=0.1.$

(iii) $P(A\cup B^c)=P(A)+P(B^c)-P(A\cap B^c)=0.3+0.4-0.1=0.6.$

(iv) $P(A^c\cup B^c)=P[(A\cap B)^c]=1-P(A\cap B)=1-0.2=0.8.$ #

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2) (i) $P(A\cap B)=P(\emptyset)=0$, so that for independence, we must have $P(A)P(B)=0$, which happens if at least one of $P(A)$, $P(B)$ is 0.

(ii) Here $A\cap B=\emptyset$, so that $P(A\cap B)=0$, whereas
$P(A)P(B)=\frac{1}{8}\times\frac{3}{8}=\frac{3}{24}\neq 0$, and $A$, $B$ are dependent.

Indeed, $P(A|B)=\frac{P(A\cap B)}{P(B)}=0\neq\frac{1}{8}=P(A)$. #

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3) (i) Since $2x-x^2=x(2-x)\geq 0$ for $0<x\leq 2$, we have to have $c\int_0^2(2x-x^2)\,dx=1$. But $\int_0^2(2x-x^2)\,dx=x^2\Big|_0^2-\frac{x^3}{3}\Big|_0^2$

$=\frac{4}{3}$, so that $\frac{4c}{3}=1$ and $c=3/4$.

(ii) $P(\frac{1}{2}<X<\frac{3}{2})=\frac{3}{4}\left(x^2\Big|_{1/2}^{3/2}-\frac{x^3}{3}\Big|_{1/2}^{3/2}\right)=\frac{3}{4}\times\frac{11}{12}=\frac{11}{16}=0.6875.$

(iii) $F(x)=\begin{cases}0, & x\leq 0\\ 3/4\left(\int_0^x(2t-t^2)\,dt\right), & 0<x<2\\ 1, & x\geq 2\end{cases}=\begin{cases}0, & x\leq 0\\ \frac{x^2}{4}(3-x), & 0<x<2\\ 1, & x\geq 2.\end{cases}$ #

4) (i) $P(E) = P(E|A)P(A) + P(E|B) + P(E|C)P(C)$

$= (0.8 \times 0.2) + (0.5 \times 0.5) + (0.3 \times 0.4) = 0.16 + 0.25 + 0.12 = 0.53.$

(ii) $P(A|E) = \dfrac{P(E|A)P(A)}{P(E)} = \dfrac{0.16}{0.53} = \dfrac{16}{53} \simeq 0.30\dot{2}.$ #

5) (i) $8! = 1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 = 40{,}320.$

(ii)

$$\begin{array}{cccccccc} \times & \times & \times & \times & \times & \times & \times & \times \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{array}$$

The 5 women can sit next to each other in 4 ways (1 2 3 4 5, 2 3 4 5 6, 3 4 5 6 7, 4 5 6 7 8), and for each such seating, there are 5! different arrangements for the 5 women and 3! for the 3 men. Thus, the answer is: $4 \times 5! \times 3! = 4 \times 120 \times 6 = 2{,}880.$

(iii)

$$\underbrace{\times \quad \times} \quad \underbrace{\times \quad \times} \quad \underbrace{\times \quad \times} \quad \underbrace{\times \quad \times}$$

Divide the 8 places in 4 adjacent pairs as indicated above, and then the required number is:

$\binom{4}{1} \times 2! \times \binom{3}{1} \times 2! \times \binom{2}{1} \times 2! \times \binom{1}{1} \times 2! = 4 \times 3 \times 2 \times 1 \times 2^4$

$= 24 \times 16 = 384.$ #