Statistics 120 Spring Quarter 2008

Answers to HW # 9 (Chapter 4)

2.9 From the table, we have:

$$P(M \cap F) = \frac{4}{16+x}, P(W \cap F) = \frac{6}{16+x},$$
$$P(M \cap S) = \frac{6}{16+x}, P(W \cap S) = \frac{x}{16+x},$$
$$P(M) = \frac{10}{16+x}, P(W) = \frac{6+x}{16+x}, P(F) = \frac{10}{16+x}, P(S) = \frac{6+x}{16+x}.$$

Then the first relation expressing independence becomes: $\frac{4}{16+x} = \frac{10}{16+x} \times \frac{10}{16+x}$ or $x = 9$. Observe that for this value of x, the remaining three relations also hold. ■

2.11 The student will pass the test if he/she answers correctly at least 5 of the last 10 questions. The probability that the student answers correctly exactly x specified of these questions is: $(0.2)^x(0.8)^{10-x}$. Since the x questions can be chosen in $\binom{10}{x}$ ways, the required probability is: $\sum_{x=5}^{10} \binom{10}{x}(0.2)^x(0.8)^{10-x} = 1 - \sum_{x=0}^{4} \binom{10}{x}(0.2)^x(0.8)^{10-x} = 1 - 0.96721 = 0.03279$. ■

2.15 Let I and H be the events of intercept and hit, respectively.

(i) Then the probability of a single missile hitting the target is:

$$P(H) = P(H|I)P(I) + P(H|I^c)P(I^c) = 0 \times \frac{1}{3} + \frac{3}{4} \times \frac{2}{3} = 0.50.$$

Therefore:

(ii) $P(\text{4 hits}) = [P(H)]^4 = (0.5)^4 = 0.0625$.

(iii) $P(\text{of at least 1 hit}) = 1 - P(\text{no hits}) = 1-(0.50)^4 = 1-0.0625 = 0.9375$.

(iv) If n missiles are fired, then $P(\text{at least 1 is not intercepted}) = 1 - P(\text{all } n \text{ are intercepted}) = 1 - (\frac{1}{3})^n$. Therefore n is defined as the smallest n for which $1 - (\frac{1}{3})^n \geq 0.95$, or $(\frac{1}{3})^n \leq 0.05$, or $n \geq -\log(0.05)/\log 3 \simeq 2.726$. Thus, $n = 3$.

(v) If n missiles are fired, then $P(\text{at least 1 hits the target}) = 1 - P(\text{none hits the target}) = 1 - (0.50)^n$. Then n is defined as the smallest n for which $1 - (0.50)^n \geq 0.99$ or $(0.50)^n \leq 0.01$, or $n \geq \frac{\log(0.01)}{\log(0.50)} \simeq 6.645$, so that $n = 7$. ■

2.17 (i) Setting $p_W = P(W_i) = 0.9$ and $p_R = P(R_j) = 0.6$, we have that the required probability is:

$P((W_1 \cap R_1^c \cap \cdots \cap R_{n-2}^c \cap R_{n-1}) \cup (W_1^c \cap W_2 \cap R_1^c \cap \cdots \cap R_{n-3}^c \cap R_{n-2})$
$\cup \cdots \cup (W_1^c \cap \cdots \cap W_{n-2}^c \cap W_{n-1} \cap R_n))$
$= P(W_1 \cap R_1^c \cap \cdots \cap R_{n-2}^c \cap R_{n-1}) + P(W_1^c \cap W_2 \cap R_1^c \cap \cdots \cap R_{n-3}^c \cap R_{n-2})$
$+ \cdots + P(W_1^c \cap \cdots \cap W_{n-2}^c \cap W_{n-1} \cap R_n)$
$= P(W_1)P(R_1^c)\ldots P(R_{n-2}^c)P(R_{n-1}) + P(W_1^c)P(W_2)P(R_1^c)\ldots$
$P(R_{n-3}^c)P(R_{n-2}) + \cdots + P(W_1^c)\ldots P(W_{n-2}^c)P(W_{n-1})P(R_n)$
(by independence of all events involved)
$= p_W(1-p_R)^{n-2}p_R + (1-p_W)p_W(1-p_R)^{n-3}p_R + \cdots$
$+ (1-p_W)^{n-2}p_W p_R$
$= p_W p_R[(1-p_R)^{n-2} + (1-p_W)(1-p_R)^{n-3} + \cdots + (1-p_W)^{n-2}]$
$= p_W p_R \sum_{i=1}^{n-1}(1-p_W)^{i-1}(1-p_R)^{(n-i)-1}$

$= (0.9 \times 0.6)\sum_{i=1}^{n-1}(0.1)^{i-1}(0.4)^{n-i-1} = 0.54\sum_{i=1}^{n-1}(0.1)^{i-1}(0.4)^{n-i-1}.$

(ii) For $n = 5$, this probability becomes:
$(0.54)[(0.4)^3 + (0.1)(0.4)^2 + (0.1)^2(0.4) + (0.1)^3]$
$= (0.54)(0.064 + 0.016 + 0.004 + 0.001)$
$= (0.54)(0.085) = 0.0459.$ ■

2.19 For $i = 1, 2, 3$, set A_i = "ith switch turns on", and E = "current flows from A to B." Then:

(i) $E = A_1 \cup A_2 \cup A_3$ and $P(E) = [P(A_1) + P(A_2) + P(A_3)] - [P(A_1 \cap A_2) + P(A_1 \cap A_3) + P(A_2 \cap A_3)] + P(A_1 \cap A_2 \cap A_3) = (p_1 + p_2 + p_3) - (p_1p_2 + p_1p_3 + p_2p_3) + p_1p_2p_3$ (by independence).

(ii) From part (i), $P(E) = 3p - 3p^2 + p^3$.

(iii) $P(E) = (0.90 + 0.95 + 0.99) - (0.90 \times 0.95 + 0.90 \times 0.99 + 0.95 \times 0.99) + 0.90 \times 0.95 \times 0.99 = 2.84 - 2.6865 + 0.84645 = 0.99995;$
$P(E) = 3 \times 0.96 - 3 \times (0.96)^2 + (0.96)^3$
$= 2.88 - 2.7648 + 0.884736 = 0.999936.$ ■