Statistics 120 Spring Quarter 2008

Answers to HW # 8 (Chapter 4)

1.5 With obvious notation, we have:

(i) $P(G_3|G_1 \cap G_2) = \frac{14}{18} = \frac{7}{9} \simeq 0.778$.

(ii) $P(D_3|G_1 \cap D_2) = \frac{3}{18} = \frac{1}{6} \simeq 0.167$.

(iii) $P(D_3|D_1 \cap G_2) = \frac{3}{18} = \frac{1}{6} \simeq 0.167$.

(iv) $P(D_3|G_1 \cap D_2 \text{ or } D_1 \cap G_2) = P(D_3|(G_1 \cap D_2) \cup (D_1 \cap G_2)) = \frac{3}{18} = \frac{1}{6} \simeq 0.167$. ■

1.9 We have: $P(A_j|A) = \frac{P(A \cap A_j)}{P(A)} = \frac{P(A|A_j)P(A_j)}{P(A)}$, where $P(A|A_j)P(A_j) = \frac{5-j}{15} \times \frac{j}{15} = \frac{j(5-j)}{225}$ and $P(A) = \sum_{j=1}^{5} P(A|A_j)P(A_j) = \sum_{j=1}^{5} \frac{j(5-j)}{225} = \frac{1}{225}(1 \times 4 + 2 \times 3 + 3 \times 2 + 4 \times 1) = \frac{20}{225}$, so that: $P(A_1|A) = \frac{4/225}{20/225} = \frac{4}{20} = \frac{1}{5} = 0.2$, $P(A_2|A) = \frac{6/225}{20/225} = \frac{6}{20} = \frac{3}{10} = 0.3$, $P(A_3|A) = P(A_2|A) = 0.3, P(A_4|A) = P(A_1|A) = 0.2, P(A_5|A) = 0$. ■

1.13 With obvious notation, we have:

(i) $P(+) = P(+|D)P(D) + P(+|D^c)P(D^c) = 0.95 \times 0.0001 + 0.05 \times 0.9999 = 0.000095 + 0.049995 = 0.05009$.

(ii) $P(D|+) = \frac{P(+|D)P(D)}{P(+)} = \frac{0.000095}{0.05009} \simeq 0.002$. ■

1.17 With obvious notation, we have:

(i) $P(B) = P(B|I)P(I) + P(B|II)P(II)$
$= \frac{b_1}{b_1+r_1} \times \frac{1}{2} + \frac{b_2}{b_2+r_2} \times \frac{1}{2} = \frac{1}{2}(\frac{b_1}{b_1+r_1} + \frac{b_2}{b_2+r_2})$.

(ii) $P(I|B) = \frac{P(B|I)P(I)}{P(B)} = \frac{\frac{b_1}{b_1+r_1} \times \frac{1}{2}}{\frac{1}{2}(\frac{b_1}{b_1+r_1} + \frac{b_2}{b_2+r_2})} = \frac{b_1(b_2+r_2)}{b_1(b_2+r_2)+b_2(b_1+r_1)}$.

(iii) $P(B) = \frac{1}{2}(\frac{36}{48} + \frac{60}{84}) = \frac{1}{2}(\frac{3}{4} + \frac{5}{7}) = \frac{41}{56} \simeq 0.732$,
$P(I|B) = \frac{3/8}{41/56} = \frac{21}{41} \simeq 0.512$. ■

1.19 (i) Let $p_n = P(A|B)$ when the number of answers is n. Then $p_n = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$, where $P(B|A)P(A) = 1 \times p = p$, and $P(B) = P(B|A)P(A) + P(B|A^c)P(A^c) = p + \frac{1}{n}(1-p)$, so that $p_n = \frac{p}{p+\frac{1-p}{n}}$.

(ii) When the number of answers is $n+1$, the respective probability p_{n+1} is given by: $p_{n+1} = \frac{p}{p+\frac{1-p}{n+1}}$. Then $p_{n+1} > p_n$ is equivalent to $\frac{p}{p+\frac{1-p}{n+1}} > \frac{p}{p+\frac{1-p}{n}}$ or $\frac{1}{n} > \frac{1}{n+1}$ (since $p < 1$), which is true. So, p_n is, indeed, increasing in n.

(iii) The larger n is, the more extensive the list of answers the student will have to choose from. Thus, if the student answers the question correctly, this would imply that it is more likely that he/she, actually, did the homework. In other words, p_n should increase with n. ■

1.21 Let A_i be the event that the exam is passed the ith time, $i = 1, 2, 3$. Then:

(i) $P(A_2) = P(A_1^c \cap A_2) = P(A_2|A_1^c)P(A_1^c) = 0.8 \times (1 - 0.7) = 0.24$.

(ii) $P(A_3) = P(A_1^c \cap A_2^c \cap A_3) = P(A_3|A_1^c \cap A_2^c)P(A_2^c|A_1^c)P(A_1^c) = 0.9 \times (1 - 0.8) \times (1 - 0.7) = 0.9 \times 0.2 \times 0.3 = 0.054$.

(iii) $P(\text{passing the exam}) = P[A_1 \cup (A_1^c \cap A_2) + (A_1^c \cap A_2^c \cap A_3)]$
$= P(A_1) + P(A_1^c \cap A_2) + P(A_1^c \cap A_2^c \cap A_3)$
$= P(A_1) + P(A_2) + P(A_3)$
$= 0.7 + 0.24 + 0.054 = 0.994$. ■