

Answers to HW #6 (Chapter 3)

2.8 Denote by  $A, B$ , and  $C$  the events that a student reads news magazines  $A, B$ , and  $C$ , respectively. Then the required probability is  $P(A^c \cap B^c \cap C^c)$ . However,

$$\begin{aligned} P(A^c \cap B^c \cap C^c) &= P((A \cup B \cup C)^c) = 1 - P(A \cup B \cup C) \\ &= 1 - [P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) \\ &\quad - P(B \cap C) + P(A \cap B \cap C)] \\ &= 1 - (0.20 + 0.15 + 0.10 - 0.05 - 0.04 - 0.03 + 0.02) \\ &= 1 - 0.35 = 0.65. \quad \blacksquare \end{aligned}$$

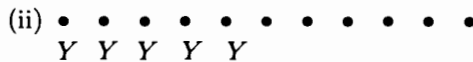
2.9 (i) Denoting by  $A$  and  $B$ , respectively, that the student is admitted at college  $A$ , and is rejected by college  $B$ , the required probability is:  $P(A \cup B^c) = P(A) + P(B^c) - P(A \cap B^c) = p_1 + 1 - p_2 - P(A \cap B^c)$ . However,  $P(A \cap B^c) = P((A^c \cup B)^c) = 1 - P(A^c \cup B) = 1 - p_3$ , so that  $P(A \cup B^c) = p_1 + 1 - p_2 - 1 + p_3 = p_1 - p_2 + p_3$ .

(ii)  $P(A \cup B^c) = 0.6 - 0.2 + 0.3 = 0.7. \quad \blacksquare$

2.11 In the first place, there are 11 blocks altogether and the number of ways of lining them up would be  $11!$ , if they were distinct. In order to find the number of distinct permutations, we have to divide  $11!$  by the product  $2! \times 4! \times 5!$  to obtain  $\frac{11!}{2! \times 4! \times 5!}$ . Arguing in a similar fashion, we get:



The number of distinct arrangements is:  $\frac{9!}{4! \times 5!}$ , so that  $\frac{9!}{4! \times 5!} / \frac{11!}{2! \times 4! \times 5!} = \frac{2! \times 9!}{11!} = \frac{1}{55} \approx 0.018$ .



For each position of the yellow blocks, there are  $\frac{6!}{2! \times 4!}$  distinct arrangements of the remaining blocks. Since the yellow blocks can be placed in 7 ways, so that to be adjacent, the required probability is:  $\frac{7 \times 6!}{2! \times 4! \times 5!} / \frac{11!}{2! \times 4! \times 5!} = \frac{7 \times 5! \times 6!}{11!} = \frac{1}{66} \approx 0.015$ .



Place 2 blue blocks (any two, since they are indistinguishable) at the two ends. Then the remaining 9 blocks produce  $\frac{9!}{2! \times 2! \times 5!}$  distinct arrangements. Thus, the required probability is:

$$\frac{9!}{2! \times 2! \times 5!} / \frac{11!}{2! \times 4! \times 5!} = \frac{4! \times 9!}{2! \times 11!} = \frac{6}{55} \approx 0.109. \quad \blacksquare$$

2.13 The required probability, clearly, is:

$$\frac{\binom{10}{2} \times \binom{15}{3} \times \binom{30}{4} \times \binom{5}{1}}{\binom{60}{10}} = \frac{2,480,625}{66,661,386} \approx 0.037. \quad \blacksquare$$

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2.16 The 500 bulbs can be chosen in  $\binom{2,000}{500}$  ways, and  $x$  defective can be chosen in  $\binom{200}{x}$  ways, whereas the  $500 - x$  good bulbs can be chosen in  $\binom{1,800}{500-x}$  ways. Since the probability of having exactly  $x$  defective bulbs among the 500 chosen is:  $\binom{200}{x}\binom{1,800}{500-x}/\binom{2,000}{500}$ , the required probability is given by:  
$$\frac{1}{\binom{2,000}{500}} \sum_{x=0}^{25} \binom{200}{x} \binom{1,800}{500-x}. \blacksquare$$

2.19 (i)  $\binom{50}{5} = \frac{50!}{5!45!} = \frac{46 \times 47 \times 48 \times 49 \times 50}{2 \times 3 \times 4 \times 5} = 2 \times 10 \times 46 \times 47 \times 48 = 2,118,760.$

(ii)  $\binom{30}{3} \binom{20}{2} = \frac{30!}{3!27!} \times \frac{20!}{2!18!} = \frac{28 \times 29 \times 30}{2 \times 3} \times \frac{19 \times 20}{2} = (5 \times 28 \times 29) \times (10 \times 19) = 771,400.$

(iii)  $\frac{771,400}{2,118,760} = \frac{19,285}{52,969} \simeq 0.364 \blacksquare.$