Statistics 120 | Spring Quarter 2008

Answers to HW #6 (Chapter 3)

2.8 Denote by A, B, and C the events that a student reads news magazines A, B, and C, respectively. Then the required probability is $P(A^c \cap B^c \cap C^c)$. However,

$$\begin{aligned}
P(A^c \cap B^c \cap C^c) &= P((A \cup B \cup C)^c) = 1 - P(A \cup B \cup C) \\
&= 1 - [P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) \\
&\quad - P(B \cap C) + P(A \cap B \cap C)] \\
&= 1 - (0.20 + 0.15 + 0.10 - 0.05 - 0.04 - 0.03 + 0.02) \\
&= 1 - 0.35 = 0.65. \quad \blacksquare
\end{aligned}$$

2.9 (i) Denoting by A and B, respectively, that the student is admitted at college A, and is rejected by college B, the required probability is: $P(A \cup B^c) = P(A) + P(B^c) - P(A \cap B^c) = p_1 + 1 - p_2 - P(A \cap B^c)$. However, $P(A \cap B^c) = P((A^c \cup B)^c) = 1 - P(A^c \cup B) = 1 - p_3$, so that $P(A \cup B^c) = p_1 + 1 - p_2 - 1 + p_3 = p_1 - p_2 + p_3$.

(ii) $P(A \cup B^c) = 0.6 - 0.2 + 0.3 = 0.7$. $\blacksquare$

2.11 In the first place, there are 11 blocks altogether and the number of ways of lining them up would be 11!, if they were distinct. In order to find the number of distinct permutations, we have to divide 11! by the product $2! \times 4! \times 5!$ to obtain $\frac{11!}{2! \times 4! \times 5!}$. Arguing in a similar fashion, we get:

(i) • • • • • • • • • • •

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The number of distinct arrangements is: $\frac{9!}{4! \times 5!}$, so that $\frac{9!}{4! \times 5!} / \frac{11!}{2! \times 4! \times 5!} = \frac{2! \times 9!}{11!} = \frac{1}{55} \simeq 0.018$.

(ii) • • • • • • • • • • •

Y Y Y Y Y

For each position of the yellow blocks, there are $\frac{6!}{2! \times 4!}$ distinct arrangements of the remaining blocks. Since the yellow blocks can be placed in 7 ways, so that to be adjacent, the required probability is: $\frac{7 \times 6!}{2! \times 4!} / \frac{11!}{2! \times 4! \times 5!} = \frac{7 \times 5! \times 6!}{11!} = \frac{1}{66} \simeq 0.015$.

(iii) • • • • • • • • • • •

B B

Place 2 blue blocks (any two, since they are indistinguishable) at the two ends. Then the remaining 9 blocks produce $\frac{9!}{2! \times 2! \times 5!}$ distinct arrangements. Thus, the required probability is: $\frac{9!}{2! \times 2! \times 5!} / \frac{11!}{2! \times 4! \times 5!} = \frac{4! \times 9!}{2! \times 11!} = \frac{6}{55} \simeq 0.109$. $\blacksquare$

2.13 The required probability, clearly, is:

$$\frac{\binom{10}{2} \times \binom{15}{3} \times \binom{30}{4} \times \binom{5}{1}}{\binom{60}{10}} = \frac{2{,}480{,}625}{66{,}661{,}386} \simeq 0.037. \quad \blacksquare$$

2.16 The 500 bulbs can be chosen in $\binom{2,000}{500}$ ways, and x defective can be chosen in $\binom{200}{x}$ ways, whereas the $500 - x$ good bulbs can be chosen in $\binom{1,800}{500-x}$ ways. Since the probability of having exactly x defective bulbs among the 500 chosen is: $\binom{200}{x}\binom{1,800}{500-x}/\binom{2,000}{500}$, the required probability is given by: $\frac{1}{\binom{2,000}{500}}\sum_{x=0}^{25}\binom{200}{x}\binom{1,800}{500-x}$. ■

2.19 (i) $\binom{50}{5} = \frac{50!}{5!45!} = \frac{46\times 47\times 48\times 49\times 50}{2\times 3\times 4\times 5} = 2 \times 10 \times 46 \times 47 \times 48 = 2,118,760.$

(ii) $\binom{30}{3}\binom{20}{2} = \frac{30!}{3!27!} \times \frac{20!}{2!18!} = \frac{28\times 29\times 30}{2\times 3} \times \frac{19\times 20}{2} = (5\times 28\times 29)\times(10\times 19) = 771,400.$

(iii) $\frac{771,400}{2,118,760} = \frac{19,285}{52,969} \simeq 0.364$ ■.