Statistics 120 Spring Quarter 2008

Answers to HW #16 (Chapter 7)

1.1 We have:

(i) $P(X=4, Y=0) = f_{X,Y}(4,0) = 0.10.$

(ii) $P(X=5) = f_{X,Y}(5,0) + f_{X,Y}(5,1) + f_{X,Y}(5,2)$
$= 0.05 + 0.03 + 0.02 = 0.10.$

(iii) $P(Y=1) = f_{X,Y}(0,1) + f_{X,Y}(1,1) + f_{X,Y}(2,1) + f_{X,Y}(3,1) + f_{X,Y}(4,1) + f_{X,Y}(5,1)$
$= 0.015 + 0.030 + 0.075 + 0.090 + 0.060 + 0.030 = 0.30.$

(iv) $P(X \le 3, Y \ge 1) = f_{X,Y}(0,1) + f_{X,Y}(0,2) + f_{X,Y}(1,1) + f_{X,Y}(1,2) + f_{X,Y}(2,1) + f_{X,Y}(2,2) + f_{X,Y}(3,1) + f_{X,Y}(3,2)$
$= 0.015 + 0.010 + 0.030 + 0.020 + 0.075 + 0.050 + 0.090 + 0.060 = 0.35.$ ■

1.3 Since $P(X<Y) + P(Y<X) = 1$, and the p.d.f. is symmetric with respect to x and y, one would expect that $P(X<Y) = 0.5$. Indeed

$$\begin{aligned} P(X<Y) &= \int_{\Re^2}\int f_{X,Y}(x,y)\,dx\,dy = \int_0^1\int_0^y (x+y)\,dx\,dy \\ &= \int_0^1 \left(\frac{x^2}{2}\Big|_0^y + y\times x\Big|_0^y\right)dy = \frac{3}{2}\int_0^1 y^2 dy = \frac{3}{2}\times\frac{y^3}{3}\Big|_0^1 = \frac{1}{2} = 0.5. \end{aligned}$$

■

1.5 (i) $P(X \le Y \le c) = \iint_{(x\le y\le c)} e^{-x-y}\,dx\,dy = \int_0^c\int_0^y e^{-x-y}\,dx\,dy$

$$= \int_0^c e^{-y}\left(\int_0^y e^{-x}dx\right)dy = \int_0^c e^{-y}(-e^{-x}|_0^y)dy = \int_0^c e^{-y}(1-e^{-y})dy$$

$$= \int_0^c e^{-y}dy - \int_0^c e^{-2y}dy = -e^{-y}|_0^c + \frac{1}{2}e^{-2y}\Big|_0^c = \frac{1}{2} - \frac{1}{e^c} + \frac{1}{2e^{2c}}.$$

(ii) For $c = \log 2$, we have: $P(X \le Y \le \log 2) = \frac{1}{8} = 0.125.$ ■

1.7 From

$$\begin{aligned} \int_0^c\int_0^y \frac{2}{c^2}\,dx\,dy &= \frac{2}{c^2}\int_0^c\int_0^y dx\,dy = \frac{2}{c^2}\int_0^c y\;dy = \frac{2}{c^2}\times\frac{y^2}{2}\Big|_0^c \\ &= \frac{2}{c^2}\times\frac{c^2}{2} = 1, \end{aligned}$$

we have that c can be any positive constant. ■

1.10 We have:
For $0 < x \le 1$, $\int_{1-x}^{2-x} cx\;dy = cx$, and for $1 < x \le 2$, $\int_0^{2-x} cx\;dy = cx(2-x)$. Since $\int_0^1 cx\,dx = \frac{c}{2}$ and $\int_1^2 cx(2-x)\;dx = \frac{2c}{3}$, we have $\frac{c}{2} + \frac{2c}{3} = 1$ and $c = \frac{6}{7}$. ■