Statistics 120 Spring Quarter 2008

Answers to HW #15 (Chapter 6)

2.13 Clearly,

(i)
$$\begin{aligned} P(X < c) &= P(X \le c) = 2 - 9[1 - P(X \le c)] \\ &= -7 + 9P(X \le c), \quad \text{so that} \\ P(X \le c) &= \frac{7}{8} \text{ or } P\left(\frac{X-\mu}{\sigma} \le \frac{c-\mu}{\sigma}\right) = \frac{7}{8} = 0.875, \end{aligned}$$

and hence (from the Normal Tables) $\frac{c-\mu}{\sigma} = 1.15$ or $c = \mu + 1.15\sigma$.

(ii) Here $c = 5 + 1.15 \times 2 = 7.30$. ■

2.15 Since $X \sim N(\mu, \sigma^2)$ (approximately) with $\mu = 105$ and $\sigma = 20$, we have:

(i) $P(X \ge 50) = P(\frac{X-\mu}{\sigma} \ge \frac{50-105}{20}) = P(Z \ge -2.75) = P(Z \le 2.75) = \Phi(2.75) = 0.997020$.

(ii) $P(X \le 80) = P(\frac{X-\mu}{\sigma} \le \frac{80-105}{20}) = P(Z \le -1.25) = \Phi(-1.25) = 1 - \Phi(1.25) = 1 - 0.894350 = 0.10565$.

(iii) $P(95 \le X \le 125) = P(\frac{95-105}{20} \le \frac{X-\mu}{\sigma} \le \frac{125-105}{20}) = P(-0.5 \le Z \le 1)$
$= \Phi(1) - \Phi(-0.5) = \Phi(1) - [1 - \Phi(0.5)] = \Phi(0.5) + \Phi(1) - 1 = 0.691462 + 0.841345 - 1 = 0.532807$.

Remark: In parts (i) and (ii), without changing the outcome, but in order to take into account the nature of the r.v. X, we have:
$P(X \ge 50) = 1 - P(0 < X \le 50) = 1 - \Phi(5.25) + \Phi(2.75) = \Phi(2.75)$;
$P(X \le 80) = P(0 < X \le 80) = \Phi(5.25) - \Phi(1.25) = 1 - \Phi(1.25)$. ■

2.17 Let X be the r.v. denoting the diameter of a ball bearing. Then the probability p that a ball bearing is defective is given by:

$$\begin{aligned} p &= P(X > 0.5 + 0.0006 \text{ or } X < 0.5 - 0.0006) \\ &= P(X > 0.5006 \text{ or } X < 0.4994) \\ &= P(X > 0.5006) + P(X < 0.4994) \\ &= 1 - P(X \le 0.5006) + P(X < 0.4994) \\ &= 1 - P\left(\frac{X-\mu}{\sigma} \le \frac{0.5006 - 0.5007}{0.0005}\right) + P\left(\frac{X-\mu}{\sigma} < \frac{0.4994 - 0.5007}{0.0005}\right) \\ &\simeq 1 - P(Z \le -0.2) + P(Z \le -2.6) \\ &= 1 - \Phi(-0.2) + \Phi(-2.6) \\ &= 1 - [1 - \Phi(0.2)] + 1 - \Phi(2.6) \\ &= \Phi(0.2) - \Phi(2.6) + 1 \\ &= 0.579260 - 0.995339 + 1 \\ &= 0.583921. \end{aligned}$$

Thus, if Y is the r.v. denoting the number of defective ball bearings from among those in a day's production, then $Y \sim B(n,p)$, where n is the number of ball bearings in a day's production. Then the expected number

of defective ball bearings is $EY = np$ and the expected proportion of defective ball bearings is $\frac{EY}{n} = \frac{np}{n} = p \simeq 0.583921$, or about 58%.

Remark: As in Exercise 2.15, $P(X > 0.5006) + P(X < 0.4994) = 1 - P(0 < X \leq 0.5006) + P(0 < X < 0.4994) = 1 - \Phi(-0.2) + \Phi(1001.4) + \Phi(-2.6) - \Phi(-1001.4) = 1 + \Phi(0.2) - \Phi(2.6)$. ■

2.19 (i)

$$\begin{aligned} p &= P(X \leq x_p) = P(\frac{X-\mu}{\sigma} \leq \frac{x_p - \mu}{\sigma}) \\ &= P(Z \leq \frac{x_p - \mu}{\sigma}) = \Phi(\frac{x_p - \mu}{\sigma}), \end{aligned}$$

so that $\frac{x_p - \mu}{\sigma} = \Phi^{-1}(p)$ and $x_p = \mu + \sigma\Phi^{-1}(p)$.

(ii) Since $\Phi^{-1}(0.25) = -\Phi^{-1}(0.75) = -0.675$, and $\Phi^{-1}(0.5) = 0$, we have: $x_{0.25} = \mu - 0.675\sigma$, $x_{0.50} = \mu$, and $x_{0.75} = \mu + 0.675\sigma$. ■

2.27 Since $f(x) = \frac{1}{2\alpha}$, for $-\alpha < x < \alpha$, we have:

(i) $P(-1 < X < 2) = \frac{3}{2\alpha} = 0.75$ and $\alpha = 2$.

(ii) $P(|X| < 1) = P(-1 < X < 1) = \frac{2}{2\alpha} = \frac{1}{\alpha}, P(|X| > 2) = 1 - P(|X| \leq 2) = 1 - P(-2 \leq X \leq 2) = 1 - \frac{4}{2\alpha} = 1 - \frac{2}{\alpha}$, so that $\frac{1}{\alpha} = 1 - \frac{2}{\alpha}$ and $\alpha = 3$. ■