Statistics 120 Spring Quarter 2008

Answers to HW #13 (Chapter 6)

1.13 (i) $P(X \leq 10) = 1 - P(X \geq 11) = 1 - p\sum_{x=11}^{\infty} q^{x-1} = 1 - pq^{10} \times \frac{1}{1-q} = 1 - q^{10}$.

(ii) $1 - (0.8)^{10} \simeq 1 - 0.107 = 0.893$. ■

1.14 Here X has the Geometric distribution with $p = 0.01$, so that $f(x) = (0.01)(0.99)^{x-1}, x = 1, 2, \ldots$ Then:

$$\begin{aligned} P(X \leq 10) &= 1 - P(X \geq 11) = 1 - (0.01)(0.99)^{10}[1 + 0.99 + (0.99)^2 + \cdots] \\ &= 1 - (0.01)(0.99)^{10} \times \frac{1}{1 - 0.99} = 1 - (0.99)^{10} \simeq 0.096. \quad ■ \end{aligned}$$

1.15 If X is the r.v. denoting the number of tosses to the first success, then X has the Geometric distribution with parameter $p = \frac{1}{6}$, so that $P(X = x) = \frac{1}{6}(\frac{5}{6})^{x-1}, x = 1, 2, \ldots$ Then:

(i) $P(X = 3) = \frac{1}{6}\left(\frac{5}{6}\right)^2 = \frac{25}{216} \simeq 0.116$.

(ii)
$$\begin{aligned} P(X \geq 5) &= \sum_{x=5}^{\infty} \frac{1}{6}\left(\frac{5}{6}\right)^{x-1} = \frac{1}{6}\sum_{x=5}^{\infty}\left(\frac{5}{6}\right)^{x-1} = \frac{1}{6} \times \left(\frac{5}{6}\right)^4 \times \frac{1}{1 - \frac{5}{6}} \\ &= \left(\frac{5}{6}\right)^4 \simeq 0.482. \quad ■ \end{aligned}$$

1.21 Indeed,

$$f(x+1) = e^{-\lambda}\frac{\lambda^{x+1}}{(x+1)!} = e^{-\lambda}\frac{\lambda \times \lambda^x}{x!(x+1)} = \frac{\lambda}{x+1} \times e^{-\lambda}\frac{\lambda^x}{x!} = \frac{\lambda}{x+1}f(x). \quad ■$$

1.27 (i) The distribution of X is Hypergeometric with $m = 15$ and $n = 10$, so that:

$$f(x) = \frac{\binom{15}{x}\binom{10}{15-x}}{\binom{25}{15}}, \quad x = 0, 1, \cdots, 15.$$

(ii) $P(X \geq 10) = \sum_{x=10}^{15} \frac{\binom{15}{x}\binom{10}{15-x}}{\binom{25}{x}}$.

(iii) This probability is 0, since there are only 10 specimens from the rock R_2. ■

1.31 Let X_1 and X_2 be the r.v.'s denoting the number of voters favoring propositions #1 and #2, respectively. Then $X_1 \sim B(2n, p_1)$ and $X_2 \sim B(2n, p_2)$, so that:

(i) $EX_1 = (2n)p_1 = 2np_1$, $EX_2 = (2n)p_2 = 2np_2$.

(ii) $P(X_1 \leq n) = \sum_{x=0}^{n} \binom{2n}{x} p_1^x (1 - p_1)^{2n-x}$.

(iii) $P(X_2 \leq n) = \sum_{x=0}^{n} \binom{2n}{x} p_2^x (1 - p_2)^{2n-x}$.

(iv) $EX_1 = 24 \times 0.3125 = 7.5$, $EX_2 = 24 \times 0.4375 = 10.5$, $P(X_1 \leq 12) = 0.9835$, $P(X_2 \leq 12) = 0.7953$ (from the Binomial Tables). ■