Department of Statistics

Statistics 120

Spring Quarter 2008

SAMPLE FINAL EXAM

1. [9] Two manufacturing processes I and II produce certain items, which are either good (event G) or defective (event D). The proportions of the defective items are 2% and 3% for processes I and II, respectively.
   A fair dice is rolled. If an odd number appears, an item from among those produced by process I is selected at random, otherwise an item from those produced by process II is selected at random.
   Compute the following probabilities:
   (i) P(D); (ii) P(I|D); (iii) P(II|D).

2. [9] Consider the following function $f(x) = x^3 + x^2 - 1$, $1<x<2$, and:
   (i) Show that $f(x)>0$, $1<x<2$.
   (ii) Is $f(x)$ a p.d.f? Justify your answer.
   (iii) If not, modify $f(x)$ to a function $g(x)$ so that $g(x)$ is a p.d.f..

3. [20] The (discrete) r.v.'s X and Y have a joint p.d.f. given by the following table:

y \ x	1	2	3	
1	1/16	3/16	2/16	
2	4/16	1/16	5/16	
				1

   (i) Compute the marginal p.d.f.'s $f_X(1)$, $f_X(2)$, $f_X(3)$; $f_Y(1)$, $f_Y(2)$.
   (ii) Show that $EX=34/16$, $EX^2=84/16$, $\sigma^2(X)=188/16^2$; $EY=26/16$, $EY^2=46/16$, $\sigma^2(Y)=60/16^2$.
   (iii) Also, show that $E(XY)=55/16$.
   (iv) From parts (ii) and (iii), compute the Cov(X,Y) and the $\rho(X,Y)$.

4. [18] In reference to Problem 3, compute the following quantities:
   (i) $P(X=1|Y=1)$, $P(X=2|Y=1)$, $P(X=3|Y=1)$; $P(X=1|Y=2)$, $P(X=2|Y=2)$, $P(X=3|Y=2)$.
   (ii) $E(X|Y=1)$, $E(X|Y=2)$.
   (iii) $P(X\geq 2|Y=1)$, $P(X\geq 2|Y=2)$.

5. [5] In reference to Problem 3, decide weather or not the r.v.'s X and Y are independent in any way you wish, but justify your answer.

6. [18] Items produced by a certain manufacturing process are either good (G), or defective but usable (U), or outright defective (D). The respective proportions are 0.8, 0.15, and 0.05. From a large lot, 10 items are selected at random, and let X_G, X_U, and X_D be the r.v.'s denoting the umbers of good, defective but usable, and outright defective items among the 10 selected.
   Compute the following quantities:
   (i) $P(X_G=7, X_U=2, X_D=1)$.
   (ii) $P(X_U=2)$.
   (iii) $P(X_G=7, X_D=1|X_U=2)$.

7. [12] The r.v. X has p.d.f. $f(x)=\sqrt{2/\pi}\,x^2\exp(-x^2/2)$, $x>0$, and let $Y=2^{-1}mX^2$, $m>0$.
   (i) Express the d.f. F_Y of Y in terms of the d.f. F_X of X.
   (ii) Differentiate $F_Y(y)$ with respect to y in order to find the p.d.f. $f_Y(y)$ of Y.
   (iii) If $g(t)=\{\Gamma(\alpha)\beta^{\alpha}\}^{-1}t^{\alpha-1}e^{-t/\beta}$, $t>0$ ($\alpha>0$, $\beta>0$), and $\Gamma(3/2)=(1/2)\Gamma(1/2)=\sqrt{\pi}/2$, bring $f_Y(y)$ under the form of $g(t)$
   (iv) Do you happen to recognize the p.d.f. f_Y?

   Remark: The r.v. X represents the velocity of a molecule of mass m, and the r.v. Y stands for the kinetic energy of the molecule.

8. [9] Refer to Problem 6, and suppose that 100 items are selected at random and independently from a very large lot of such items. Then the probability, p say, of an item to be defective is p=0.05. Define the r.v.'s X_i, $i=1, \ldots, 100$ as follows: $X_i=1$ if the ith item is defective, and $X_i=0$ otherwise, so that X_i, $i=1, \ldots, 100$ are independent r.v. distributed as B(1, 0.05).
   (i) What are EX_i, $\sigma^2(X_i)$, and $\sigma(X_i)$? (Just state them, you don't have to derive them).
   (ii) Set $S_{100}=\sum_{i=1}^{100} X_i$. What is the distribution of the r.v. S_{100} and what are the ES_{100} and $\sigma^2(S_{100})$?
   (iii) Use the CLT in order to find an approximate value to the probability $P(3 \le S_{100} \le 8)$.

   Remark: Use the attached Normal Tables.