Brief Solutions

1. For complete data ($X = 10$), the fisher information is

$$i(\mu) = -\frac{\partial}{\partial \mu^2}(\log f)|_{x=10} = 1$$

in which f is the p.d.f. of $\mathcal{N}(\mu, 1)$

For incomplete data case, the p.d.f. $g(H(X) = 10) = P(X > 10) = S(10; \mu)$

$$i_{H(X)}(\mu) = -\frac{\partial}{\partial \mu^2}(\log g)|_{H(x)=10} = \frac{(S')^2 - S^{''}S}{(S)^2}$$

in which S is the survival function of $\mathcal{N}(\mu, 1)$

The information for incomplete data is always smaller than 1 because of the censorship (loss of information). Numeric result needed.

2. (a) $E(T) = \frac{1}{\lambda}$
(b) $E(T|T > 10) = \frac{1}{\lambda} + 10$
(c) $F(t|T > 10) = 1 - e^{-(t-10)\lambda} \quad (t > 10)$, so the p.d.f is $f(t|T > 10) = \lambda e^{-(t-10)\lambda}, \quad t > 10$
(d) $f_S(s) = \frac{\lambda}{\alpha\beta} e^{\alpha/\beta s^{1/\alpha}} s^{1/\alpha - 1}$

3. (a) The MLE of a and b are solved by maximizing

$$l(a, b) = \sum \delta_i \log(aX_i + bX_i^2) - \sum(\frac{a}{2}X_i^2 + \frac{b}{3}X_i^3)$$

and we have asymptotic normality for $(\hat{a}, \hat{b})^T$

$$((\hat{a}, \hat{b})^T - (a, b)^T) \sim \mathcal{N}(0, \Sigma)$$

in which Σ^{-1} is the Hessian matrix.
The 95% confidence ellipse is given by

$$(\hat{a} - a, \hat{b} - b)\hat{\Sigma}^{-1}(\hat{a} - a, \hat{b} - b)^T < 5.99$$

in which $\hat{\Sigma}$ replaces the (a, b) in Σ by $(\hat{a}, \hat{b})$
(We can also simply find the confidence interval for a and b separately)
(b) The confidence bands for the cumulative hazard $\Lambda(t)$. The delta-method guarantees that

$$\Lambda - \hat{\Lambda} \sim \mathcal{N}\left(\mathbf{0}, \sigma_a^2 \left(\frac{\partial}{\partial a}\Lambda\right)^2 + 2\sigma_{ab}\frac{\partial}{\partial a}\Lambda\frac{\partial}{\partial b}\Lambda + \sigma_b^2 \left(\frac{\partial}{\partial b}\Lambda\right)^2\right)$$

In practice, we use $\Lambda(t; \hat{a}, \hat{b}) = \frac{\hat{a}}{2}t^2 + \frac{\hat{b}}{3}t^3$ for every time point t
(c) The confidence bands for $S(t)$ is derived from $S(t) = e^{-\Lambda(t)}$

4. The MLE of β is solved by maximizing

$$l(\beta; X) = -\sum \beta\delta_i Z_i - \sum X_i e^{-\beta Z_i}$$

The MLE should performs much better than β_{OLS}

5. A brief way to test: split the data set $\{X_i\}$ by the quantiles. To calculate the expected observation in each segment, we have (assuming know distribution of censoring time C_i as G)

$$P(X < t) = 1 - P((T \wedge C) > t) = 1 - (1 - F_T(t))(1 - G_C(t))$$

in which F(t) is the corresponding c.d.f to Λ_0
χ^2-test can be applied then.

6. Omitted.