Inferences Based on a Single Sample: Tests of Hypothesis (continued)

III. Small-Sample Tests for $\mu$ when $\sigma$ is unknown

Recall that when $\sigma$ is unknown and the population is normal, then

$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

is distributed as Student’s $t$ with $(n-1)$ degrees of freedom.

To test $H_0: \mu = \mu_0$, we compute

and reject $H_0$ in favor of

$$H_a: \mu < \mu_0 \text{ if } t < -t_\alpha$$

$$H_a: \mu > \mu_0 \text{ if } t > t_\alpha$$

$$H_a: \mu \neq \mu_0 \text{ if } t < -t_{\alpha/2} \text{ or } t > t_{\alpha/2}$$
Example: Lifetime of tires

For a random sample of 10 tires, the average lifetime is 41 thousand miles with standard deviation 3.59 (thousand miles). Test $H_0: \mu = 42$ vs. $H_a: \mu < 42$ at 0.05 level of significance.

The p-value:
IV. Large-Sample Tests for Population Proportion $p$

Recall that for large $n$ (that is, $p^\pm \pm \sqrt{p(1-p)/n}$ lies in the interval 0 to 1), the sample proportion

$$p^\pm = \frac{X}{n}$$

is approximately normal with mean $p$ and standard deviation $\sqrt{p(1-p)/n}$. Then,

$$z = \frac{(p^\pm - p)}{\sqrt{p(1-p)/n}}$$

is approximately standard normal.

To test $H_0 : p = p_0$, we compute

and reject $H_0$ in favor of

$$H_a : p < p_0 \text{ if } z < -z_\alpha$$

$$H_a : p > p_0 \text{ if } z > z_\alpha$$

$$H_a : p \neq p_0 \text{ if } z < -z_{\alpha/2} \text{ or } z > z_{\alpha/2}$$
Example: Common Cold

In a random sample of 250 students from a large class of STA 13, 30 showed symptoms of common cold. Test $H_0: p = 0.10$ vs. $H_a: p > 0.10$ at 0.05 level of significance.

The p-value: