Discrete Probability Distributions

I. Random Variables

A random variable is a variable whose numerical values are determined by the outcomes of a random experiment.

Examples:

Types of random variables:

1. Discrete.
2. Continuous.

II. Probability Distribution of a Discrete Random Variable

The probability distribution of a discrete random variable $X$ is represented by a probability function $p(x)$ defined as:

$$p(x) = P[X = x]$$

Example: Cost of health care

<table>
<thead>
<tr>
<th>Charges (x)</th>
<th>p(x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clinic</td>
<td>$50</td>
</tr>
<tr>
<td>Clinic and Lab</td>
<td>$80</td>
</tr>
<tr>
<td>ER visit</td>
<td>$160</td>
</tr>
</tbody>
</table>

Properties of a probability distribution:
III. Mean and Variance of Discrete Random Variables

Let X be a discrete random variable. The *expected value (mean)* of X is defined as:

\[ \mu = E(X) = \sum x \cdot p(x) \]

**Example:** Cost of health care

*Variance* of a random variable X is defined as

\[ \sigma^2 = E(X - \mu)^2 = \sum (X - \mu)^2 \cdot p(x) \]

**Example:** Cost of health care

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IV. The Binomial Distribution

a. Consider a simple experiment with two possible outcomes:

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Success (S)</th>
<th>Failure (F)</th>
</tr>
</thead>
</table>

Examples:
b. *Binomial Experiment:* An experiment with the following characteristics:

1. Each trial has 2 possible outcomes, success and failure.
2. For any trial \( P(\text{Success}) = p \) and \( P(\text{Failure}) = 1 - p \).
3. The trials are independent of each other.

**Examples:**

c. The *binomial random variable*

Consider a binomial experiment. Define \( X \) as

\[ X = \text{Number of successes in } n \text{ trials.} \]

\( X \) is a discrete random variable. The probability function of \( X \) is:
**Example 1:** Toss a coin 6 times, find probability of 2 heads.

**Example 2:** Suppose 8 out of 20 students in an elementary school class in Davis develop influenza, whereas 20% of elementary school students nationwide develop influenza. Are there an unusually high number of cases in this class?

d. Using the Binomial Table (Table II, page 563). The table gives

\[ P( X \leq k ) = P(0) + P(1) + P(2) + \ldots + P(k) \]
e. For the binomial distribution

\[ \text{Mean: } \mu = np \]
\[ \text{Variance: } \sigma^2 = np(1-p) \]

**Example 1:** Toss a coin 6 times.

**Example 2:** Influenza