14.2 (a) We first find \( k = 3 \), \( \bar{y}_1 = 27 \), \( \bar{y}_2 = 15 \), \( \bar{y}_3 = 18 \). Thus,

\[
\begin{align*}
\text{Obs.} & \quad \text{Grand mean} & \quad \text{Tr. Effect} \\
\begin{bmatrix} 35 & 24 & 28 & 21 \\ 19 & 14 & 14 & 13 \\ 21 & 16 & 21 & 14 \end{bmatrix} & \quad \begin{bmatrix} 20 & 20 & 20 & 20 \\ 20 & 20 & 20 & 20 \end{bmatrix} & \begin{bmatrix} 7 & 7 & 7 & 7 \\ -5 & -5 & -5 & -5 \end{bmatrix} \\
\begin{bmatrix} 16 & 21 & 19 & 14 & 14 & 13 & 21 & 16 & 21 & 14 \end{bmatrix} & \begin{bmatrix} 20 & 20 & 20 & 20 & 20 & 20 & 20 & 20 & 20 & 20 \end{bmatrix} & \begin{bmatrix} -2 & -2 & -2 & -2 \end{bmatrix}
\end{align*}
\]

Residuals

\[
\begin{bmatrix} y_i - \bar{y} \\
8 & -3 & 1 & -6 \\
+ & 4 & -1 & -1 & -2 \\
3 & -2 & 3 & -4 
\end{bmatrix}
\]

(b) Treatment SS = \( 4(7)^2 + 4(-5)^2 + 4(-2)^2 = 312 \)

Residual SS = \( 8^2 + (-3)^2 + \ldots + (-4)^2 = 170 \)

Total SS = \( (35 - 20)^2 + (24 - 20)^2 + (28 - 20)^2 + \ldots + (14 - 20)^2 = 482 \)

(c) Error d.f. = \( \sum n_i - k = 4 + 4 + 4 - 3 = 9 \)

Treatment d.f. = \( k - 1 = 3 - 1 = 2 \)

(d) The analysis-of-variance table is

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>d.f.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>312</td>
<td>2</td>
</tr>
<tr>
<td>Error</td>
<td>170</td>
<td>9</td>
</tr>
<tr>
<td>Total</td>
<td>482</td>
<td>11</td>
</tr>
</tbody>
</table>

14.4 The analysis-of-variance table is

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>d.f.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>34</td>
<td>5</td>
</tr>
<tr>
<td>Error</td>
<td>( 58 = 92 - 34 )</td>
<td>20 (= 25 - 5)</td>
</tr>
<tr>
<td>Total</td>
<td>92</td>
<td>25</td>
</tr>
</tbody>
</table>
14.7 The overall mean is
\[ y = \frac{20 \times 94.4 + 20 \times 92.9 + 20 \times 75.5}{60} = 87.6 \]

Treatment SS = 20 \((94.4 - 87.6)^2 + 20 \((92.9 - 87.6)^2 + 20 \((75.5 - 87.6)^2 = 4,414.80 \]

SSE = 19 \((58.4)^2 + 19 \((54.2)^2 + 19 \((38.1)^2 = 148,196.39 \]

ANOVA Table

<table>
<thead>
<tr>
<th>Source</th>
<th>Sum of Squares</th>
<th>d.f.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td>4,414.80</td>
<td>2</td>
</tr>
<tr>
<td>Error</td>
<td>148,196.39</td>
<td>57</td>
</tr>
<tr>
<td>Total</td>
<td>152,611.9</td>
<td>59</td>
</tr>
</tbody>
</table>

14.10 From the F-table, \( F_{0.10} (2, 57) = 2.16 \). We observe that

\[ F = \frac{\text{Treatment SS} / (k - 1)}{\text{SSE} / (n-k)} = \frac{12/2}{104/41} = 3.65 \]

Consequently, we reject the hypothesis of equal treatment means, at level \( \alpha = 0.10 \).

14.13 We are to test the null hypothesis \( H_0: \mu_1 = \mu_2 = \mu_3 \) versus the alternative hypothesis that the means are not all equal. Given \( \alpha = 0.05 \), the rejection region is determined by the value \( F_{0.05} (2, 9) = 4.26 \) obtained from the F-table. From Exercise 14.2, the observed value of \( F \) is

\[ F = \frac{\text{Treatment SS} / (k - 1)}{\text{SSE} / (n-k)} = \frac{312/2}{170/9} = 8.26 \]

Consequently, we reject the null hypothesis that the means are equal at the \( \alpha = 0.05 \) level of significance.

14.20 (a) We chose \( \alpha = 0.06 \). With \( m = \left( \begin{array}{c} 3 \\ 2 \end{array} \right) = 3 \), we have \( \alpha = \frac{0.01}{2m} = 0.01 \). From Appendix Table 4 with d.f. = 57, we interpolate \( t_{0.01} = 2.388 \). From the ANOVA table, \( s = 50.9896 \). Hence simultaneous 94% confidence intervals for the differences are given by

\[ \mu_1 - \mu_2: \quad (94.4 - 92.9) \pm 2.388 \times 50.9896 \sqrt{1/20 + 1/20} = 1.5 \pm 38.904 \]

or \((-37.60, 39.64)\)

\[ \mu_1 - \mu_3: \quad (94.4 - 75.5) \pm 2.388 \times 50.9896 \sqrt{1/20 + 1/20} = 18.9 \pm 38.904 \]

or \((-19.60, 57.40)\)

\[ \mu_2 - \mu_3: \quad (92.9 - 75.5) \pm 2.388 \times 50.9896 \sqrt{1/20 + 1/20} = 17.4 \pm 38.504 \]

or \((-21.10, 55.90)\)

(b) Even if the observed differences were statistically significant, they could have been systematically caused by uncontrolled variables. Also, the causal relation could conceivably be in the reverse direction.