1. A manager believes that technology companies that have words such as cyber, link, and Web in their name have lesser growth over the past year than technology companies that do not. To test this belief, a random sample of technology companies with and without these words in their title were sampled and their percentage growth rates were recorded:

With: 16.50 10.25 11.50 10.75 11.00 6.60 10.40
Without: 18.75 11.00 9.75 10.50 10.00

Do the data support the manager's belief?

(4) a. State the null and alternative hypotheses.

(12) b. Test at 0.05 level of significance. What assumptions are you making in this hypothesis testing?

(4) c. Find the p-value.

\[ H_0: \mu_1 = \mu_2 \quad H_1: \mu_1 < \mu_2 \]

\[ \bar{x}_1 = 11 \quad \bar{x}_2 = 12 \]

\[ s_1 = 2.91 \quad s_2 = 3.80 \]

\[ \frac{s_2^2}{s_1^2} = \frac{3.80^2}{2.91^2} = 1.70 < 2 \quad \text{use pooled procedure} \]

\[ s_p^2 = \frac{(7-1) \times 2.91^2 + (5-1) \times 3.80^2}{7 + 5 - 2} = 10.87 \]

\[ t = \frac{(\bar{x}_1 - \bar{x}_2) - 0}{\sqrt{s_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}} = -0.54 \]

\[ \text{df} = 7 + 5 - 2 = 10 \]

\[ t_{0.05} = 1.812 \quad \text{Reject } H_0 \text{ at } 0.05 \text{ level} \]

\[ \text{p-value} > 0.25 \]
2. The following data were obtained from a random sample of eight oak trees on their age in years (x), and the diameter of their trunk in inches (y).

<table>
<thead>
<tr>
<th>x</th>
<th>22</th>
<th>14</th>
<th>31</th>
<th>36</th>
<th>9</th>
<th>41</th>
<th>19</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>20</td>
<td>16</td>
<td>30</td>
<td>39</td>
<td>17</td>
<td>42</td>
<td>23</td>
<td>21</td>
</tr>
</tbody>
</table>

(4) a. Write down the regression model and explain its components.

b. Obtain the estimated least-squares regression equation, and explain the meaning of the estimated slope in the context of this problem.

(8) c. Test the null hypothesis that the correlation coefficient is equal to zero versus the alternative hypothesis that the coefficient correlation is positive at 0.01 level of significance and find the p-value.

(8) d. Find the 95% prediction interval for the diameter of the trunk when the age of the tree is 30 years. Explain the meaning of this prediction interval.

(5) e. Find the coefficient of determination and explain its meaning.

\[ y = \beta_0 + \beta_1 x + \epsilon \]

\[ b_1 = 0.846, \quad b_0 = 5.21 \]

\[ \hat{y} = 5.21 + 0.846 x \]

\[ H_0: \rho = 0, \quad H_A: \rho > 0 \]

\[ r = 0.961, \quad df = 8 - 2 = 6 \]

\[ t = 8.512, \quad p \text{-value} = .000 \]

Reject \( H_0 \)

\[ \hat{y} = 31.2, \quad t_{10} = 2.447 \]

\[ S_e = 2.97 \]

95%. \( P_{\text{red. nd.}} \) [23.36, 39.04]

\[ r^2 = 0.924 \]

92.4 %

A Brief solution of the regression example:

\[ x \text{-bar} = 2, \quad y \text{-bar} = 14, \sum (x - \bar{x})^2 = 8, \sum (y - \bar{y})^2 = 144, \sum (x - \bar{x})(y - \bar{y}) = 32, b_1 = 4, b_0 = 6, \]

\[ y^* = 6 + 4x, \quad SS(\text{resid}) = 16, \quad Se = 1.633, \quad Sbr = .5774, \quad t = 6.9276, \quad r = .9428. \]

Data:

<table>
<thead>
<tr>
<th>x</th>
<th>2</th>
<th>1</th>
<th>3</th>
<th>1</th>
<th>4</th>
<th>2</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>16</td>
<td>9</td>
<td>17</td>
<td>12</td>
<td>22</td>
<td>13</td>
<td>8</td>
<td>15</td>
</tr>
</tbody>
</table>
3. A digital instrument is used to test the acid level of a particular drug under three different temperatures. Six replicates of each experiment are run in the laboratory and the results are shown below:

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Replicate 50</th>
<th>Replicate 60</th>
<th>Replicate 80</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>155</td>
<td>75</td>
<td>70</td>
</tr>
<tr>
<td>2</td>
<td>130</td>
<td>80</td>
<td>82</td>
</tr>
<tr>
<td>3</td>
<td>188</td>
<td>136</td>
<td>80</td>
</tr>
<tr>
<td>4</td>
<td>126</td>
<td>106</td>
<td>98</td>
</tr>
<tr>
<td>5</td>
<td>110</td>
<td>174</td>
<td>104</td>
</tr>
<tr>
<td>6</td>
<td>160</td>
<td>150</td>
<td>82</td>
</tr>
</tbody>
</table>

The between sum of square is 10,474 and the within sum of squares is 12,670.

(4) a. Write down the analysis of variance model and explain its components.

(10) b. Construct the ANOVA table.

(5) c. Perform a global F test at 0.05 level and explain the result.

(3) d. Find the p-value for the F-test.

(8) e. Construct a 95% confidence interval for the difference between means in temperatures 50 and 80. Explain the meaning of this confidence interval.

\[ y_{ij} = \mu + \tau_i + \varepsilon_{ij} \]

\( \begin{array}{c|c|c|c|c}
\text{Source} & \text{df} & \text{SS} & \text{MS} & \text{F} \\
\hline
\text{Temp} & 2 & 10474 & 5237 & 6.20 \\
\text{Error} & 15 & 12670 & 845 & \text{ } \\
\hline
\text{Total} & 17 & 23144 & \text{ } & \text{ } \\
\end{array} \)

\( F = \frac{\text{MS(\text{Treatment})}}{\text{MSE}} = 6.20 \quad \text{df} = (2, 15) \)

\( F_{0.05} = 3.68 \quad \text{Reject } H_0 \)

\( p\text{-value} = P[F > F_0] \)

\( 0.1 < p\text{-value} < 0.02 \)

\( \hat{\mu}_1 = 144.8 \quad \hat{\mu}_3 = 86.00 \quad S_{\text{pooled}} = \sqrt{\text{MSE}} = \sqrt{845} \)

\( t_{12} = 2.131 \)

95\% CI: \( 58.8 \pm 35.76 \)

or \([23.04, 94.56]\)
4. A study was conducted to observe the relationship between the emotional stability and talent. One hundred individuals were selected randomly and the following cross-classification was obtained.

<table>
<thead>
<tr>
<th>Emotional Stability</th>
<th>Stable</th>
<th>Unstable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Talent</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Artist</td>
<td>10</td>
<td>16</td>
</tr>
<tr>
<td>Musical genius</td>
<td>20</td>
<td>14</td>
</tr>
<tr>
<td>Math</td>
<td>30</td>
<td>10</td>
</tr>
</tbody>
</table>

(2) a. State the null and alternative hypotheses in words.
(10) b. Use the chi-squares procedure to test the hypothesis stated in (a) at 0.01 level.
(3) c. What is the p-value?

a. \( H_0: \) Emotional Stability is independent of Talent
\( H_A: \) \( H_0 \) is not true

\[ \chi^2 = 8.795 \]
\( df = 2 \)

Critical value = 9.21

\( \text{Not reject } H_0 \)

\[ 0.01 < \text{p-value} < 0.025 \]