Practice Midterm I

Show all work and explain your answers (succinctly!) for full credit. You are free to use one standard, 8½ × 11 inch piece of paper, front and back, of handwritten notes; no calculators or other materials are permitted. Problems are not necessarily ordered by increasing difficulty. Manage your time well, and leave difficult problems for last. Simplify your numerical answers as much as possible.

Good luck!
1. [20 points] Let $A, B$ be mutually exclusive events, with $P(A) = 0.3$ and $P(B^c) = 0.4$.

   (i) Draw a venn diagram for the probability experiment

   (ii) Find $P(A \cup B)$.

   (iii) Suppose that a third event, $C$, is mutually exclusive with $A$, but also mutually exclusive with $B$. At most what can be the probability of $C$?
2. [20 points total] Emma has entered a poker tournament, but does not know whether Jenna will enter. Emma has probability \( \frac{2}{3} \) of winning if Jenna does not play, but only \( \frac{1}{8} \) if Jenna does. If the probability that Jenna plays is \( \frac{1}{5} \), find

(i) the probability that Emma wins the tournament,
(ii) the probability that Jenna played given that Emma wins.
3. [20 points total] Suppose that a pair of six-sided dice is rolled. Let $X$ be the number of dots appearing on top of the first die, $Y$ be the number on the second, and $Z = X - Y$.

(i) What possible values can $Z$ take on?
(ii) Draw the probability histogram for the distribution of $Z$.
(iii) What is the expected value of $Z$?
(iv) What is the probability that $Z$ is a multiple of two?
4. [20 points total] Suppose that out of ten apples at the grocery store, five are bruised. Suppose further that three of the ten are chosen at random.

   (i) Describe the simple events $S$. How many are there?
   (ii) What is the probability that all three of the chosen apples are bruised?
5. [20 points total] Suppose the probability of a student’s spending 5 or more years in college is \( \frac{1}{5} \). A man and woman start college in the same year.

(i) What is the probability that the man will spend four or fewer years in college?

(ii) What is the probability that both the man and the woman will spend four or fewer years in college? (Assume they are unrelated and their lengths of time in college are independent of each other.)

(iii) What is the probability that one or the other or both will spend 5 years or more in college?