2.1 (i) The r.v. X takes on the values: 0, 1, 2, 3.
(ii) Its p.d.f. is: \( f(0) = \frac{1}{4} = 0.125, f(1) = \frac{2}{4} = 0.5 \).
(iii) \( P(X \geq 2) = f(2) + f(3) = \frac{3}{4} = 0.75, P(X \leq 2) = 1 - P(X > 2) = 1 - f(3) = 1 - \frac{1}{4} = 0.875. \)

2.2 (i) For \( 0 < x \leq 1, f(x) = \frac{d}{dx}(x^2 - x^2 + x) = 3x^2 - 2x + 1, \) so that \( f(x) = 3x^2 - 2x + 1, 0 < x \leq 1 \) (and 0 otherwise).
(ii) \( P(X > \frac{1}{2}) = \int_{\frac{1}{2}}^{\infty} (3x^2 - 2x + 1) dx = \frac{3}{2} \left( \frac{1}{2} \right)^{2/3} + \frac{1}{2} \left( \frac{1}{2} \right) = \frac{3}{8} = 0.375. \) Alternatively, \( P(X > \frac{1}{2}) = P(X > \frac{1}{2}) = 1 - F(\frac{1}{2}) - F(\frac{1}{2}) = 1 - \frac{3}{8} = 0.625. \)

2.3 (i) For \( 0 < x \leq 1, f(x) = \int_{\frac{1}{2}}^{\infty} (\frac{1}{2} + x) dx = \frac{1}{2} x + \frac{x}{3} \), thus,
\[
F(x) = \begin{cases} 
0, & x < 0 \\
-\frac{3}{2} x^2 + \frac{5}{3} x, & 0 \leq x \leq 1 \\
1, & x > 1.
\end{cases}
\]

2.6 (i) We need two relations which are provided by: \( \int_{0}^{1} (cx + d) dx = 1 \) and \( \int_{0}^{1} (cx + d) dx = 1/3, \) or: \( c + 2d = 2 \) and \( 9c + 12d = 8, \) and hence \( c = -\frac{2}{3}, d = \frac{5}{3}. \)
(ii) For \( 0 \leq x \leq 1, F(x) = \int_{0}^{x} \left(-\frac{1}{2} t + \frac{5}{3} \right) dt = -\frac{3}{2} x^2 + \frac{5}{3} x. \)

2.11 (i) From \( \int_{c}^{1} c(1 - x^2) dx = c(x|_{1} - \frac{x^3}{3}|_{1}) = c(2 - \frac{2}{3}) = 1, \) we get \( c = 3/4. \)
(ii) \( P(-0.9 < X < 0.9) = \int_{-0.9}^{0.9} \frac{3}{4} (x - 0.6) - \frac{1}{2} (0.6) dx = \frac{3 - 1.24}{4} = 0.625 = 0.9855. \)

2.18 (i) Since \( \int_{0}^{\infty} e^{x} e^{-cx} = -cx e^{-cx} \bigg|_{0}^{\infty} - e^{-cx} \bigg|_{0}^{\infty} = 1 \) for all \( c > 0, \) the given function is a p.d.f. for all \( c > 0. \)
(ii) From part (i),
\[
P(X \geq 1) = -c x e^{-cx} \bigg|_{0}^{\infty} - e^{-cx} \bigg|_{0}^{\infty} = c(t - e^{-ct} + e^{-ct}) = \frac{c(t + 1)}{e^{ct}}.
\]
(iii) Here \( c(t + 1) = 0.2 \times 11 = 2.2, ct = 0.2 \times 20 = 2, \) so that \( \frac{ct}{e^{ct}} = \frac{0.2}{0.297} \approx 0.297. \)

2.19 (i) \( \sum_{c=1}^{5} cx = c(1 + 2 + 3 + 4 + 5) = 15c = 1, \) so that \( c = 1/15. \)
(ii) \( P(X \leq 3) = \frac{1}{15}(1 + 2 + 3) = \frac{6}{15} = \frac{2}{5} = 0.4, P(2 < X \leq 4) = \frac{1}{15}(2 + 3 + 4) = \frac{9}{15} = \frac{3}{5} = 0.6. \)
3.3 Here: (i) \( P(\text{blonde}) = \frac{50}{60} = \frac{5}{6} = 0.4 \), and
(ii) \( P(\text{blonde and blue eyes}) = \frac{P(\text{blonde and blue eyes})}{P(\text{blue eyes})} = \frac{15/60}{19/60} = \frac{15}{19} \approx 0.7714. \)

3.4 In obvious notation, we have:
(i) \( P(b_1|h_1) = \frac{P(b_1, h_1)}{P(h_1)} = \frac{0.30}{0.4} = 0.75 \approx 0.778. \)
(ii) \( P(b_1 | h_2) = P(b_1 | h_2) = 0.16 \), so
(iii) \( P(b_1) = P(b_1 | h_1) + P(b_1 | h_2) = 0.30 + 0.16 = 0.46. \)

3.5 With obvious notation, we have:
(i) \( P(G_3 | G_1 \cap G_2) = \frac{P(G_3)}{P(G_1) P(G_2)} = \frac{1.62}{0.2 \times 0.778} = 0.778. \)
(ii) \( P(D_3) | G_1 \cap G_2) = \frac{P(D_3)}{P(D_1) P(D_2) P(D_3)} = \frac{1.62}{0.3 \times 0.16 \times 0.16} = 0.46. \)
(iii) \( P(D_3) | G_1 \cap G_2) = \frac{P(D_3)}{P(D_1) P(D_2) P(D_3)} = \frac{1.62}{0.3 \times 0.16 \times 0.16} = 0.46. \)
(iv) \( P(D_3 | G_1 \cap G_2) = \frac{P(D_3 | G_1) P(G_1)}{P(D_3 | G_1) P(G_1) + P(D_3 | G_2) P(G_2)} = \frac{1.62}{1.62 + 0.46} = 0.778. \)

3.9 We have: \( P(A_i | A) = \frac{P(A_i A_j)}{P(A)} = \frac{P(A_i | A) P(A)}{P(A)} = \sum_{j=1}^{n} P(A_i | A) P(A_j) = \sum_{j=1}^{n} \frac{0.30}{0.20} = \frac{1}{5} = 0.2, \)

so that the required probability is: \( \frac{1.62}{0.20} = 8.1 \approx 0.167. \)

3.13 With obvious notation, we have:
(i) \( P(+) = P(+) P(D) + P(+) P(D) = 0.95 \times 0.001 + 0.05 \times 0.9999 = 0.00095 + 0.049995 = 0.05095. \)
(ii) \( P(D | +) = \frac{P(D | +) P(D)}{P(D | +) P(D) + P(D | -) P(D)} \approx 0.002. \)

3.19 (i) Let \( p_n = P(A | B) \) when the number of answers is \( n \). Then
\( p_n = \frac{P(A | B)}{P(B)} = \frac{P(A | B) P(B)}{P(B)} \), where \( P(B | A) \) \( P(A) = 1 \times p \cdot p \), and
\( P(B) = P(B | A) P(A) = P(B | A) P(A) = p + \frac{1}{2} (1 - p) \), so that \( p_n = \frac{p}{p + \frac{1}{2} - p}. \)
(ii) When the number of answers is \( n + 1 \), the respective probability \( p_{n+1} \) is given by: \( p_{n+1} = \frac{?}{?} \). Then \( p_{n+1} \) is equivalent to
\( \frac{?}{?} \) or \( \frac{?}{?} \) (since \( p < 1 \), which is true. So, \( p_n \) is, indeed, increasing in \( n \).
(iii) The larger \( n \) is, the more extensive the list of answers the student will have to choose from. Thus, if the student answers the question correctly, this would imply that it is more likely that he/she actually, did the homework. In other words, \( p_n \) should increase with \( n \).
4.1 From $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$, we get:

(i) $P(A \cup B \cup C) = 0.4 + 0.2 + 0.3 = 0.9$.

(ii) $P(A \cup B \cup C) = 0.9 - 0.4 \times 0.2 - 0.4 \times 0.3 + 0.4 \times 0.2 \times 0.3 = 0.664$.

4.5 (i) It is true that $P(A \cap B) = P(A)P(B)(\neq)$, $P(A \cap C) = P(A)P(C)(\neq)$, $P(B \cap C) = P(B)P(C)(\neq)$, but $P(A \cap B \cap C) = P(A)P(B)P(C)$, so that $A$, $B$, and $C$ are not independent; they are dependent.

(ii) $P(A \cap B \cap C) = 0.4 \times 0.2 \times 0.3 = 0.024$.

4.9 From the table, we have:

\[ P(M \cap F) = \frac{4}{16 + x}, \quad P(W \cap F) = \frac{6}{16 + x}, \]
\[ P(M \cap S) = \frac{6}{16 + x}, \quad P(W \cap S) = \frac{x}{16 + x}, \]
\[ P(M) = \frac{10}{16 + x}, \quad P(W) = \frac{6 + x}{16 + x}, \quad P(F) = \frac{10}{16 + x}, \quad P(S) = \frac{6 + x}{16 + x}. \]

Then the first relation expressing independence becomes: $\frac{4}{16 + x} = \frac{10}{16 + x} \times \frac{10}{16 + x}$, or $x = 9$. Observe that for this value of $x$, the remaining three relations also hold.

4.13 Let $I$ and $H$ be the events of intercept and hit, respectively. Then the probability of a single missile hitting the target is:

\[ P(H) = P(H|I)P(I) + P(H|I^c)P(I^c) = 0.5 \cdot \frac{1}{3} + \frac{2}{3} \cdot \frac{2}{3} = 0.50. \]

(i) $P(4 \text{ hits}) = (0.5)^4 = 0.0625$.

(ii) $P(\text{of at least } 1 \text{ hit}) = 1 - P(\text{no hits}) = 1 - (0.5)^4 = 0.9375$.

(iii) If $n$ missiles are fired, then $P(\text{at least } 1 \text{ is not intercepted}) = 1 - P(\text{all } n \text{ are intercepted}) = 1 - (0.5)^n$, so that $n$ is defined as the smallest $n$ for which $1 - (0.5)^n \geq 0.95$ or $(0.5)^n \leq 0.05$, or $n \geq \log(0.05)/\log 0.5 \approx 6.645$, so that $n = 7$.

(iv) If $n$ missiles are fired, then $P(\text{at least } 1 \text{ hits the target}) = 1 - P(\text{none hits the target}) = 1 - (0.5)^n$. Then $n$ is defined as the smallest $n$ for which $1 - (0.5)^n \geq 0.99$ or $(0.5)^n \leq 0.01$, or $n \geq \log(0.01)/\log 0.5 \approx 6.645$, so that $n = 7$.

4.15 With obvious notation, we have:

(i) $P(B) = P(B_1 \cap B_2 \cap B_3) + P(B_1 \cap W_2 \cap B_3) + P(W_1 \cap B_2 \cap B_3) + P(W_1 \cap W_2 \cap B_3) + P(W_1 \cap B_2 \cap W_3) + P(W_1 \cap W_2 \cap W_3) + P(B_1 \cap B_2 \cap B_3) + P(B_1 \cap W_2 \cap W_3) + P(W_1 \cap B_2 \cap W_3) + P(W_1 \cap W_2 \cap W_3)$.

(ii) By symmetry, $P(W) = \frac{m_1 n_1 + m_3 n_2 + 2m_2 n_2}{2(m_1 n_1 + m_2 n_2 + m_3 n_2)}$.

(iii) For the given values of $m_1, m_2, n_2$ and $n_2$, we have:

\[ P(B) = \frac{10 \times 25 + 15 \times 50 + 0.25 \times 25}{2 \times 25 + 15 \times 25 + 0.25 \times 25} = \frac{25 \times 61}{2 \times 25 + 15 \times 25 + 0.25 \times 25} = \frac{610}{150} = 0.506, \]
\[ P(W) = \frac{10 \times 25 + 15 \times 50 + 0.25 \times 25}{2 \times 25 + 15 \times 25 + 0.25 \times 25} = \frac{50.25}{150} = 0.335. \]

}\]