Practice Midterm (Stat 102)

1. **Narrative: Students’ GPA**

A law school administrator was interested in whether a student’s score on the entrance exam can be used to predict a student’s grade point average (GPA) after one year of law school. The administrator took a random sample of 15 students and computed the following summary information, where $x =$ entrance exam score and $y =$ GPA after one year:

- $n = 15$,
- $\sum x_i = 1293$,
- $\sum y_i = 48.58$,
- $\sum x_i y_i = 4226.2$,
- $s_x = 6.9714$ and $s_y = 0.4236$.

**a.** Find the correlation between the entrance exam score and the grade point average after one year of law school. Interpret the correlation coefficient found in the previous question.

**b.** Find the best fitting line relating grade point average after one year of law school and score on the entrance exam.

**c.** If a student scored 91 on the entrance exam, what would you predict the student’s grade point average to be after one year of law school?
3. Medical case histories indicate that different illnesses may produce identical symptom. Suppose a particular set of symptoms, which we will denote as event $H$, occurs only when any one of these illnesses: $A$, $B$, or $C$ occurs. (For the sake of simplicity, we will assume that illnesses $A$, $B$, and $C$ are mutually exclusive.) Studies show these probabilities of getting the three illnesses: $P(A) = 0.015$, $P(B) = 0.005$, and $P(C) = 0.025$. The probabilities of developing the symptoms $H$, given a specific illness, are $P(H / A) = 0.85$, $P(H / B) = 0.90$, and $P(H / C) = 0.70$.

a. Assuming that an ill person shows the symptoms $H$, what is the probability that the person has illness $A$?
4. **Narrative: Defective Bolts**

Approximately 5% of the bolts coming off a production line have serious defects. Two bolts are randomly selected for inspection.

a. Find the probability distribution for \( x \), the number of defective bolts in the sample. Hint: calculate \( P(\text{both bolts are not defective}) \), \( P(\text{one bolt is defective}) \), \( P(\text{both bolts are defective}) \) and then construct a table with two columns, one is for \( x \) and the other is its probabilities.

b. Find \( E(x) \) and \( \text{Var}(x) \).

c. Is \( x \) a binomial random variable? Justify your answer.

d. Verify your answer in part (a) and (b) using the binomial distribution.